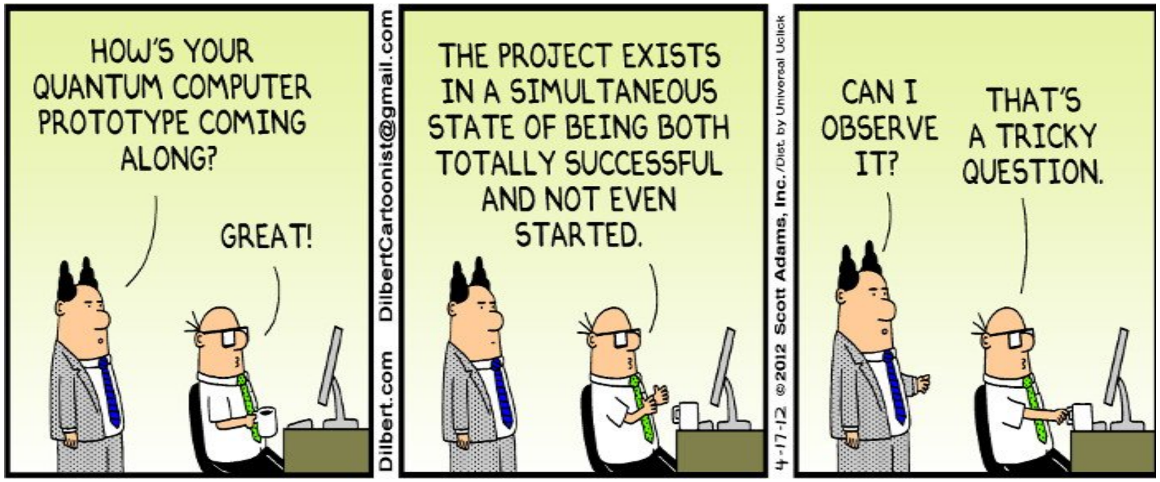
Design and Operation of the Trapped Ion Quantum Computer (TIQC)



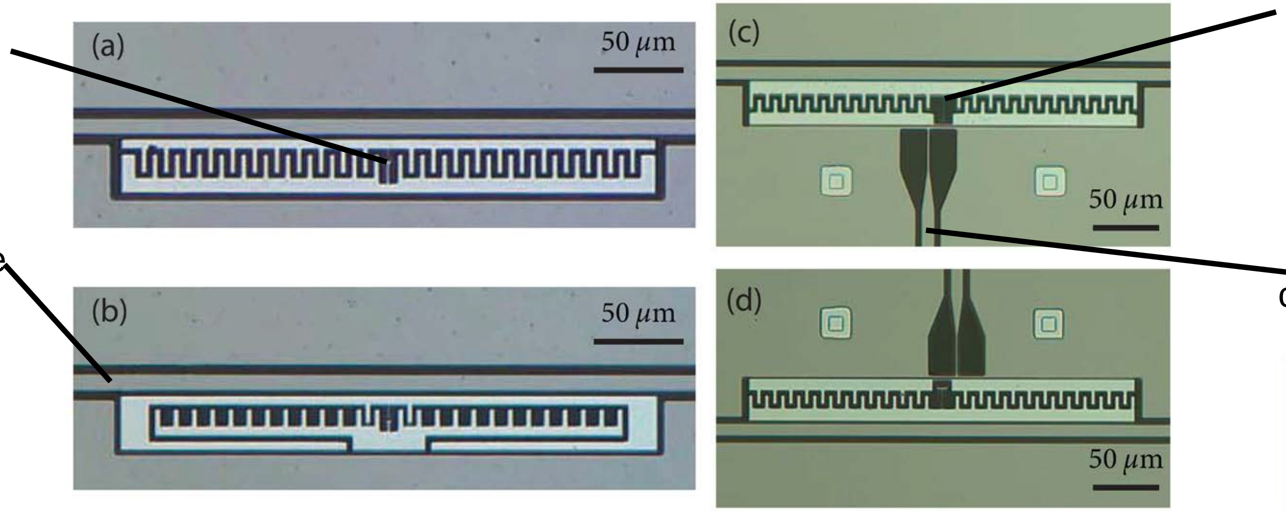
Special Topics in Computer Science:
Quantum Computing

CSC591/ECE592 – Fall 2019

(Subset of) Requirements for a Quantum Computer

- Ability to initialize the state of the qubits
- Physical system with two uniquely addressable states
- Ability to implement arbitrary rotations on the Bloch sphere
- Ability to construct a set of universal gates
- Ability to entangle two qubits
- Decoherence times sufficiently long in order to implement a circuit with enough depth to accomplish a calculation
- Ability to measure the state of a cubit

One Type of Quantum Computer Utilizes Superconducting Transmon Designs



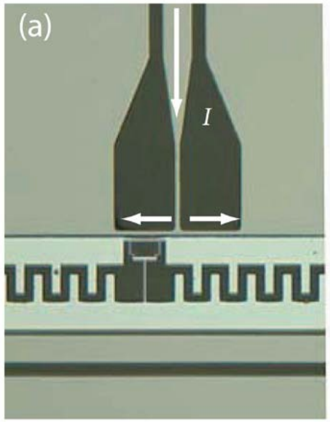
Fixed Frequency Qubits

Flux Tunable Qubits

Coplanar microwave resonator

Current control line

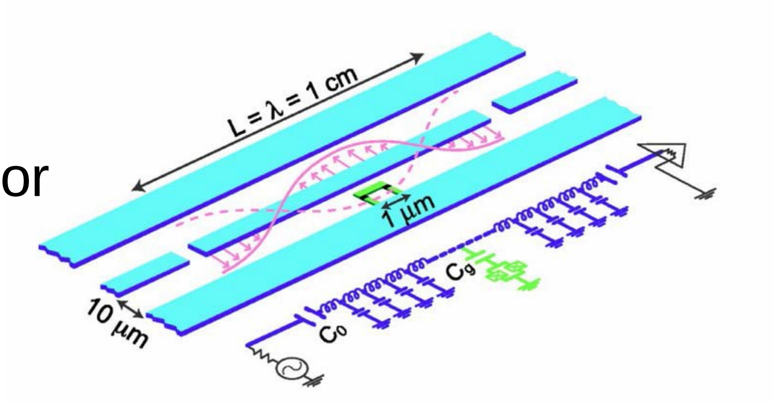
Figure 5.3: Optical images of different transmon designs. (a) Standard transmon design employed in cQED157 and on one of the qubits in cQED187. (b) Balanced transmon design used in one of the qubits in cQED187. (c) and (d) Transmon designs incorporating flux bias lines. A slightly different transmon SQUID loop design is necessary to accommodate the flux bias lines entering from the (c) bottom of the chip or from the (d) top of the chip, while preserving the same double-angle evaporation procedure.



Chow, PhD Thesis

Interactions between Qubits are Implemented by Driving a Transmon at the Other Qubit's Transmon Frequency

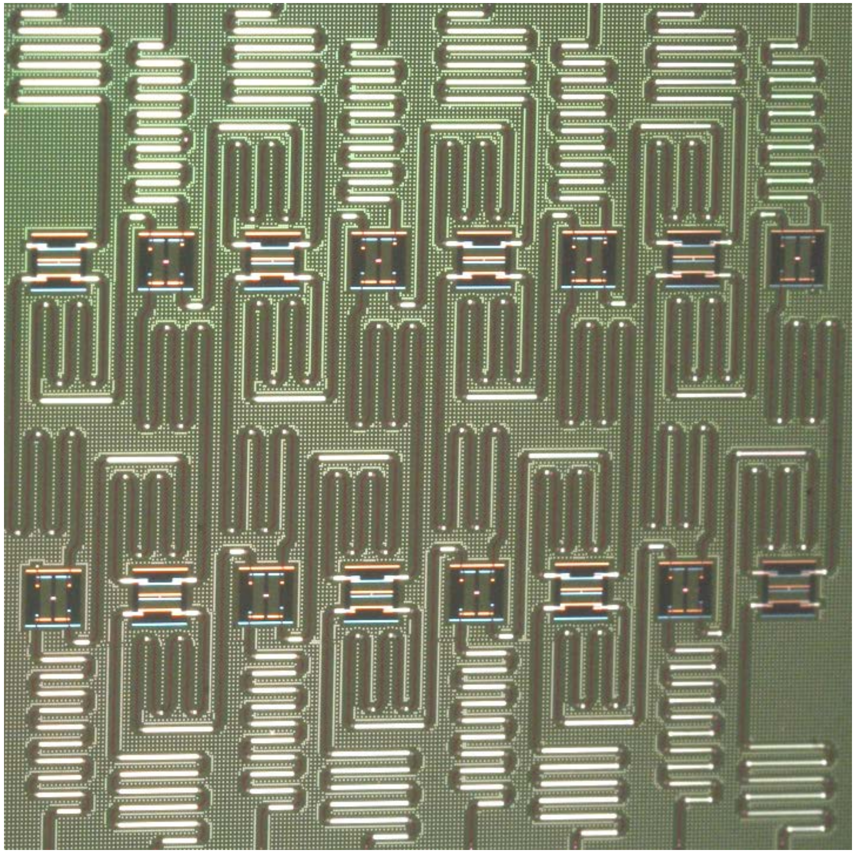
- Co-planar microstrip resonator formed by gaps in center conductor
- Important to properly choose resonator frequency with respect to transmon frequency (more to come)



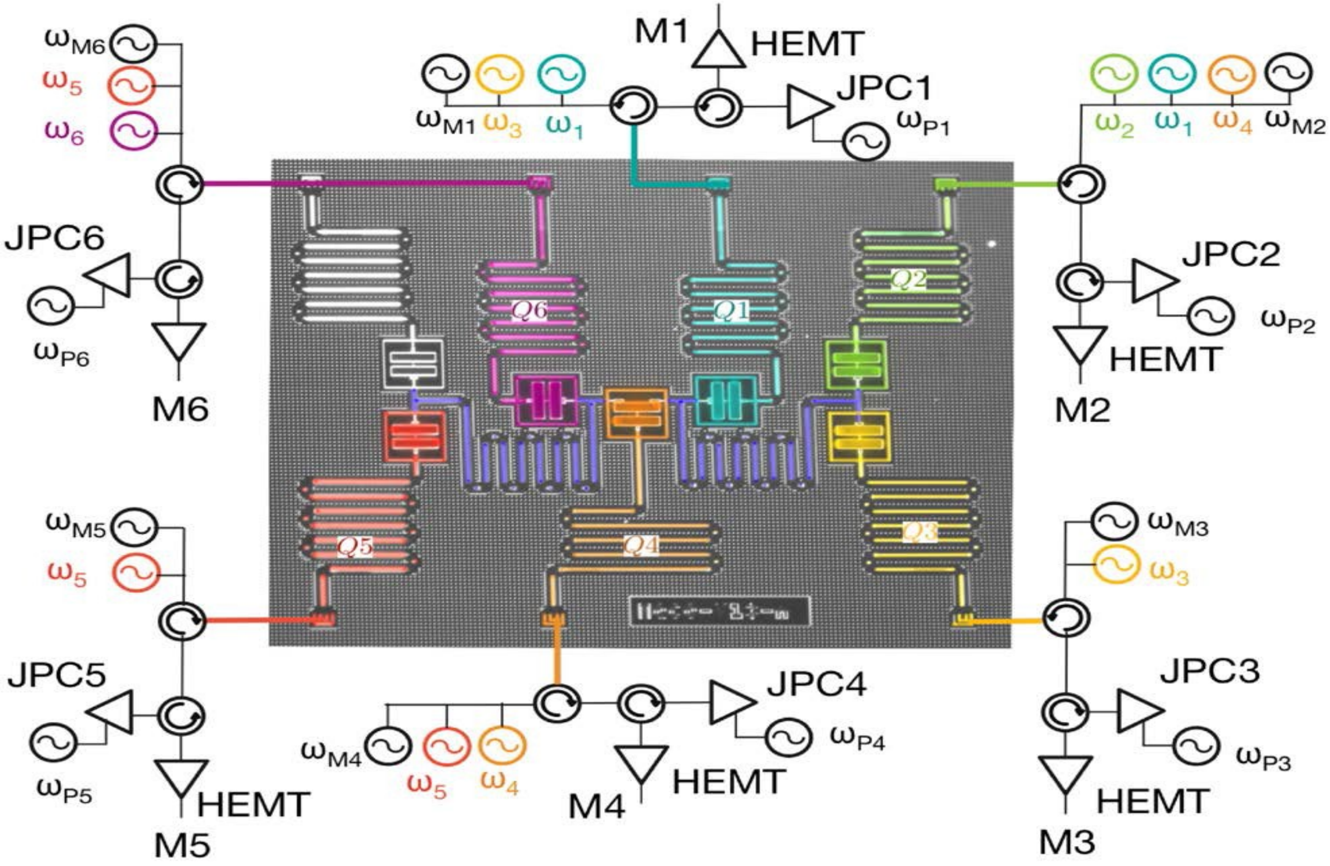
Blais, et al

- Control is achieved by injecting an RF signal from one end
- Readout is achieved by looking at either the transmitted or reflected signal

Superconducting Qubits on a Substrate



Example IBM Architecture



A. Kandala, A. Mezzacapo, K. Temme, M. Takita, M. Brink, J.M. Chow and J.M. Gambetta, “Hardware-efficient variational quantum eigensolver for small molecules and quantum magnets,” Nature, Vol. 549, p. 242 (2017).

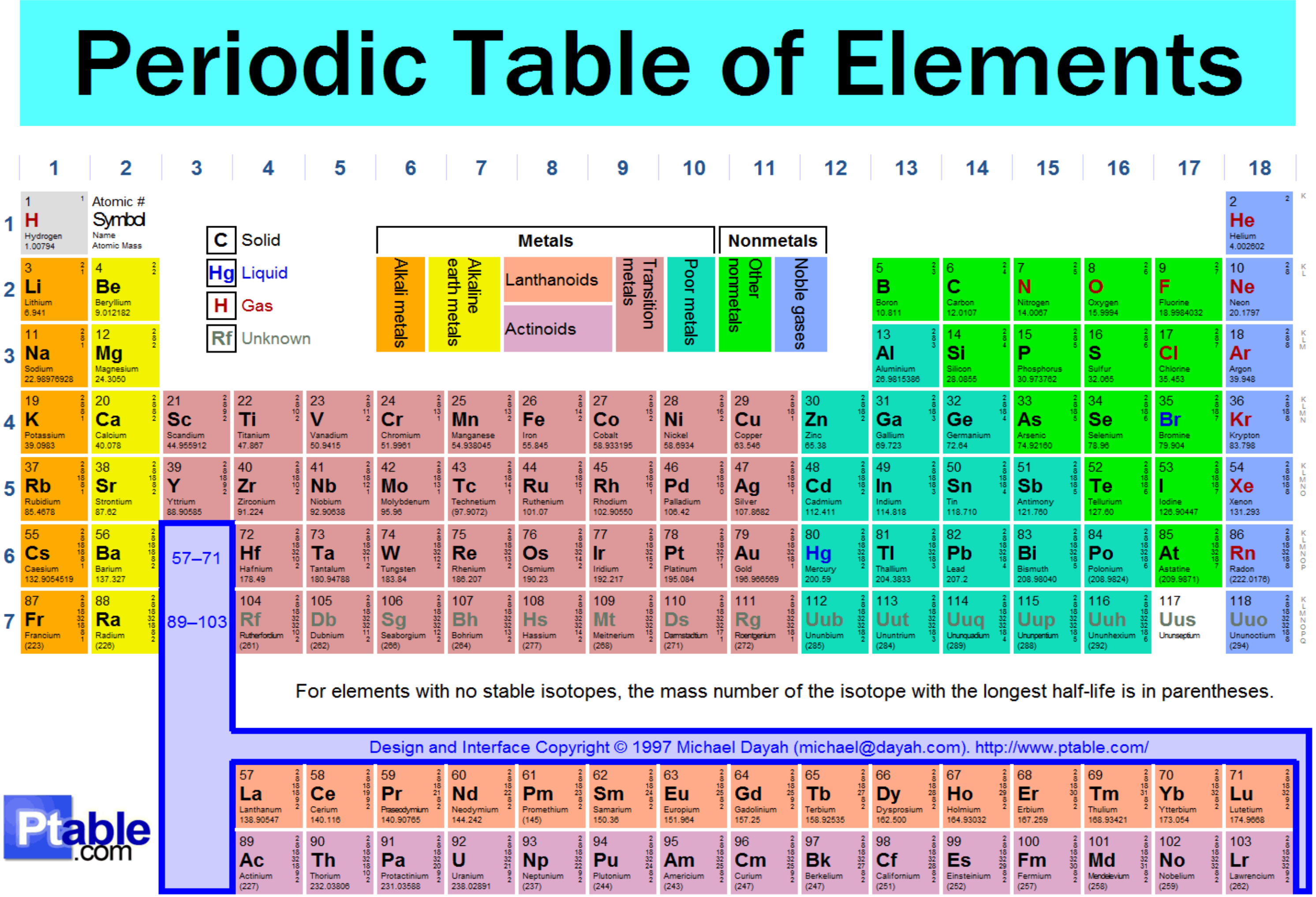
Alternative Design For Building a Quantum Computer

Trapped Ion Quantum Computer

Step 1

Select Materials That Can Emulate One and Two Qubit Operations

Start by Selecting a Material for the TIQC



Periodic Table of Elements

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 **H** Hydrogen 1.00794	Atomic # Symbol Name Atomic Mass																2 **He** Helium 4.002602
2	3 **Li** Lithium 6.941	4 **Be** Beryllium 9.012182											5 **B** Boron 10.811	6 **C** Carbon 12.0107	7 **N** Nitrogen 14.0067	8 **O** Oxygen 15.9994	9 **F** Fluorine 18.9984032	10 **Ne** Neon 20.1797
3	11 **Na** Sodium 22.98976928	12 **Mg** Magnesium 24.3050											13 **Al** Aluminium 26.9815386	14 **Si** Silicon 28.0855	15 **P** Phosphorus 30.973762	16 **S** Sulfur 32.065	17 **Cl** Chlorine 35.453	18 **Ar** Argon 39.948
4	19 **K** Potassium 39.0983	20 **Ca** Calcium 40.078	21 **Sc** Scandium 44.955912	22 **Ti** Titanium 47.867	23 **V** Vanadium 50.9415	24 **Cr** Chromium 51.9961	25 **Mn** Manganese 54.938045	26 **Fe** Iron 55.845	27 **Co** Cobalt 58.933195	28 **Ni** Nickel 58.6934	29 **Cu** Copper 63.546	30 **Zn** Zinc 65.38	31 **Ga** Gallium 69.723	32 **Ge** Germanium 72.64	33 **As** Arsenic 74.92160	34 **Se** Selenium 78.96	35 **Br** Bromine 79.904	36 **Kr** Krypton 83.798
5	37 **Rb** Rubidium 85.4678	38 **Sr** Strontium 87.62	39 **Y** Yttrium 88.90585	40 **Zr** Zirconium 91.224	41 **Nb** Niobium 92.90638	42 **Mo** Molybdenum 95.96	43 **Tc** Technetium (97.9072)	44 **Ru** Ruthenium 101.07	45 **Rh** Rhodium 102.90550	46 **Pd** Palladium 106.42	47 **Ag** Silver 107.8682	48 **Cd** Cadmium 112.411	49 **In** Indium 114.818	50 **Sn** Tin 118.710	51 **Sb** Antimony 121.760	52 **Te** Tellurium 127.60	53 **I** Iodine 126.90447	54 **Xe** Xenon 131.293
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7	87 **Fr** Francium (223)	88 **Ra** Radium (226)	89–103	104 **Rf** Rutherfordium (261)	105 **Db** Dubnium (262)	106 **Sg** Seaborgium (266)	107 **Bh** Bohrium (264)	108 **Hs** Hassium (277)	109 **Mt** Meitnerium (268)	110 **Ds** Darmstadtium (271)	111 **Rg** Roentgenium (272)	112 **Uub** Ununbium (285)	113 **Uut** Ununtrium (284)	114 **Uuq** Ununquadium (289)	115 **Uup** Ununpentium (288)	116 **Uuh** Ununhexium (292)	117 **Uus** Ununseptium	118 **Uuo** Ununoctium (294)

C Solid
Hg Liquid
H Gas
Rf Unknown

Metals: Alkali metals · Alkaline earth metals · Lanthanoids · Actinoids · Transition metals · Poor metals

Nonmetals: Other nonmetals · Noble gases

For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.

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57 **La** Lanthanum 138.90547	58 **Ce** Cerium 140.116	59 **Pr** Praseodymium 140.90765	60 **Nd** Neodymium 144.242	61 **Pm** Promethium (145)	62 **Sm** Samarium 150.36	63 **Eu** Europium 151.964	64 **Gd** Gadolinium 157.25	65 **Tb** Terbium 158.92535	66 **Dy** Dysprosium 162.500	67 **Ho** Holmium 164.93032	68 **Er** Erbium 167.259	69 **Tm** Thulium 168.93421	70 **Yb** Ytterbium 173.054	71 **Lu** Lutetium 174.9668
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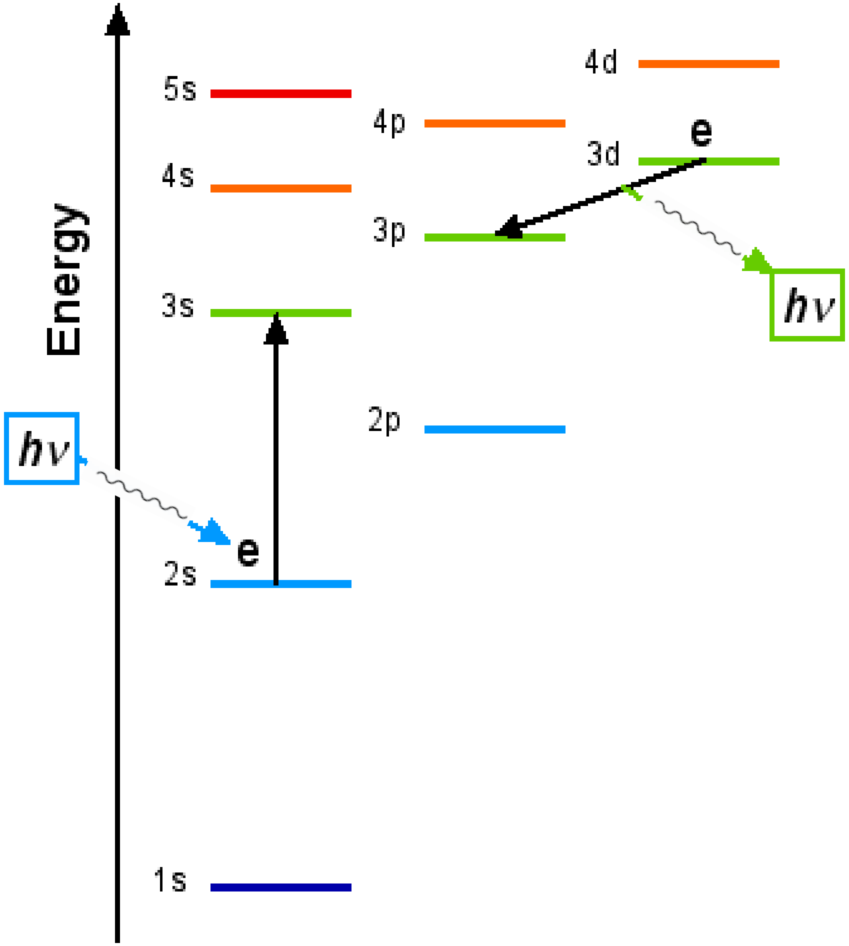
Ptable.com

QM Describes Each Element’s Atomic Structure (Energy Levels and Transitions)

- Electrons can change energy states by transitioning among different quantized energy levels
- Electrons absorb and emit discrete quantities of energy and angular momentum when undergoing these transitions



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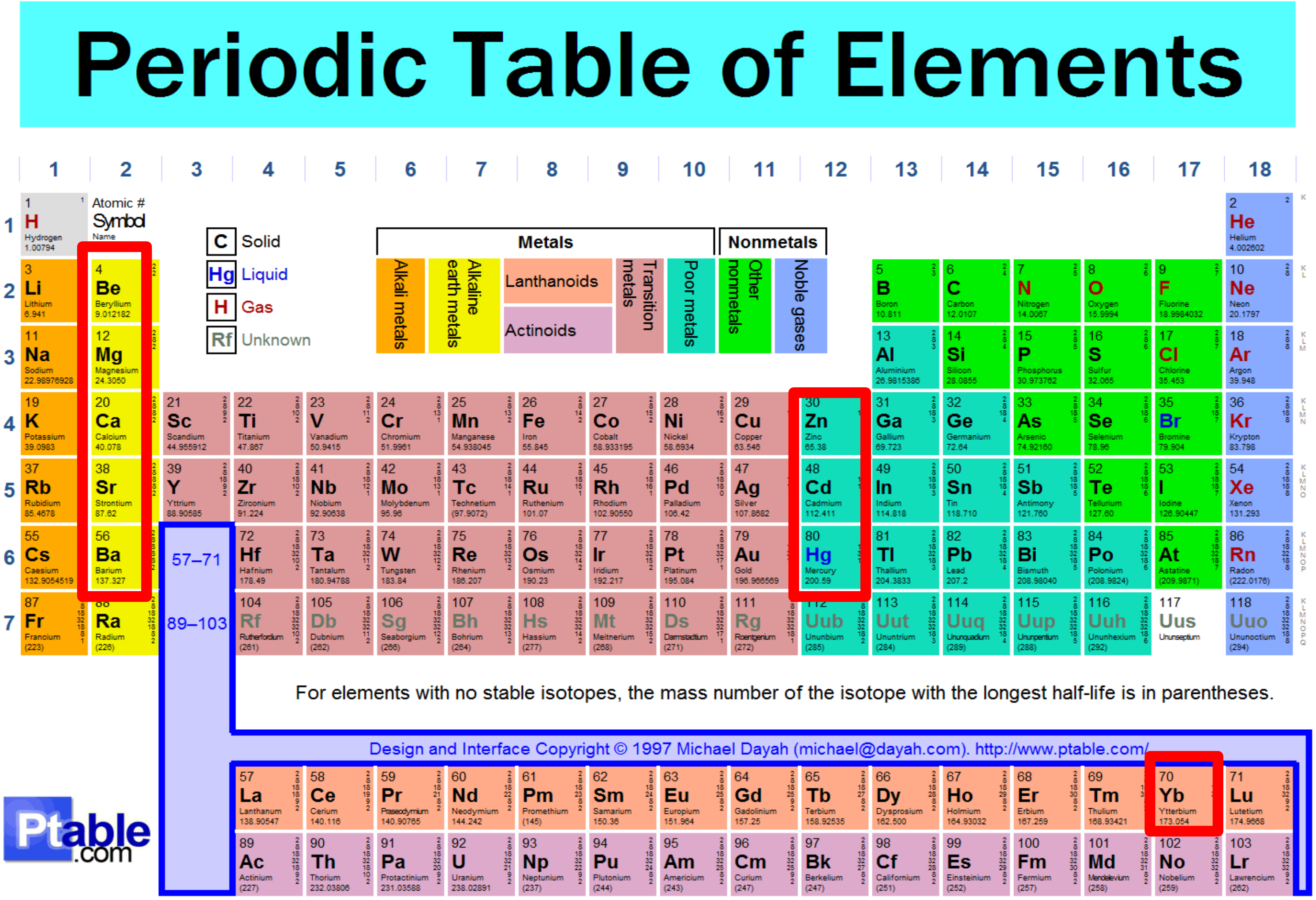

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Atomic #
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H Gas
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Metals: Alkali metals, Alkaline earth metals, Lanthanoids, Actinoids, Transition metals, Poor metals

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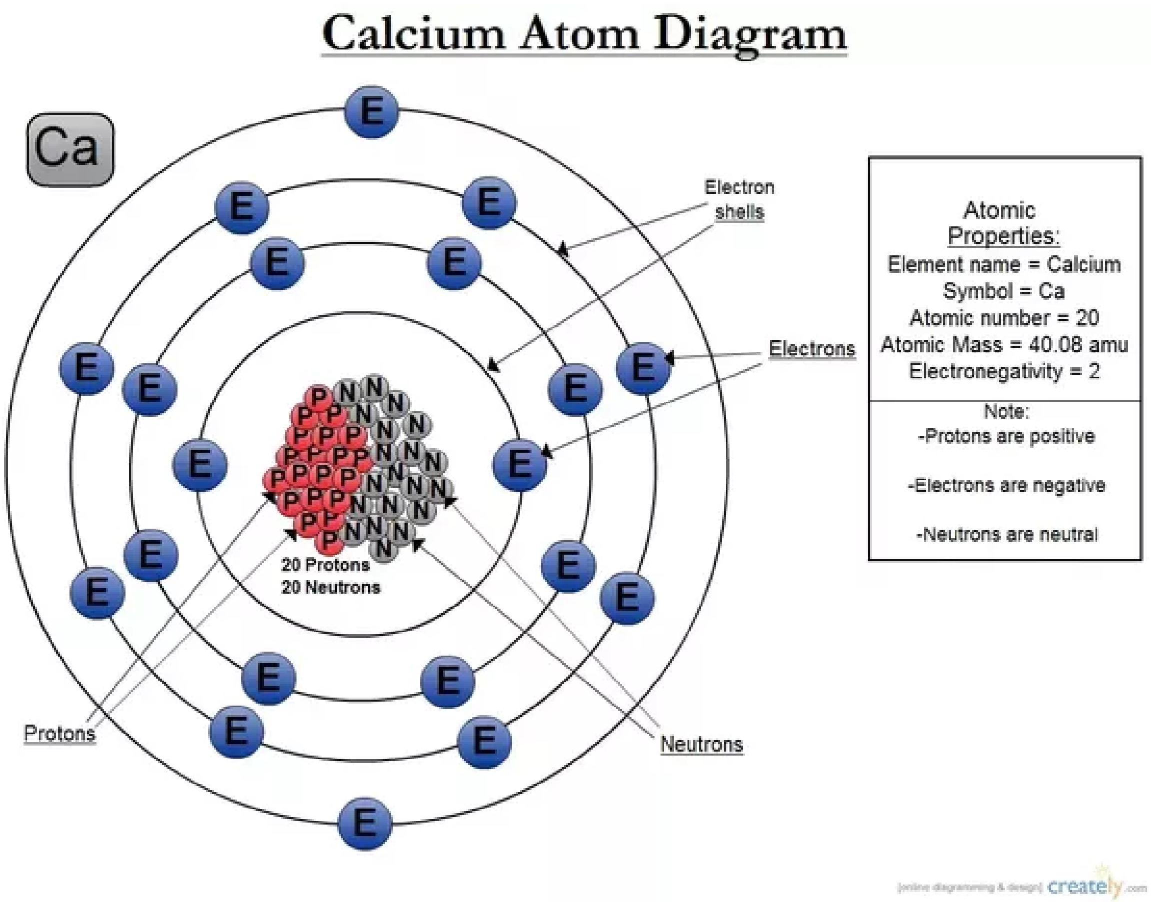
Ptable.com

Select the Calcium Atom ($^{40}_{20}$Ca)



Atomic Properties:
Element name = Calcium
Symbol = Ca
Atomic number = 20
Atomic Mass = 40.08 amu
Electronegativity = 2

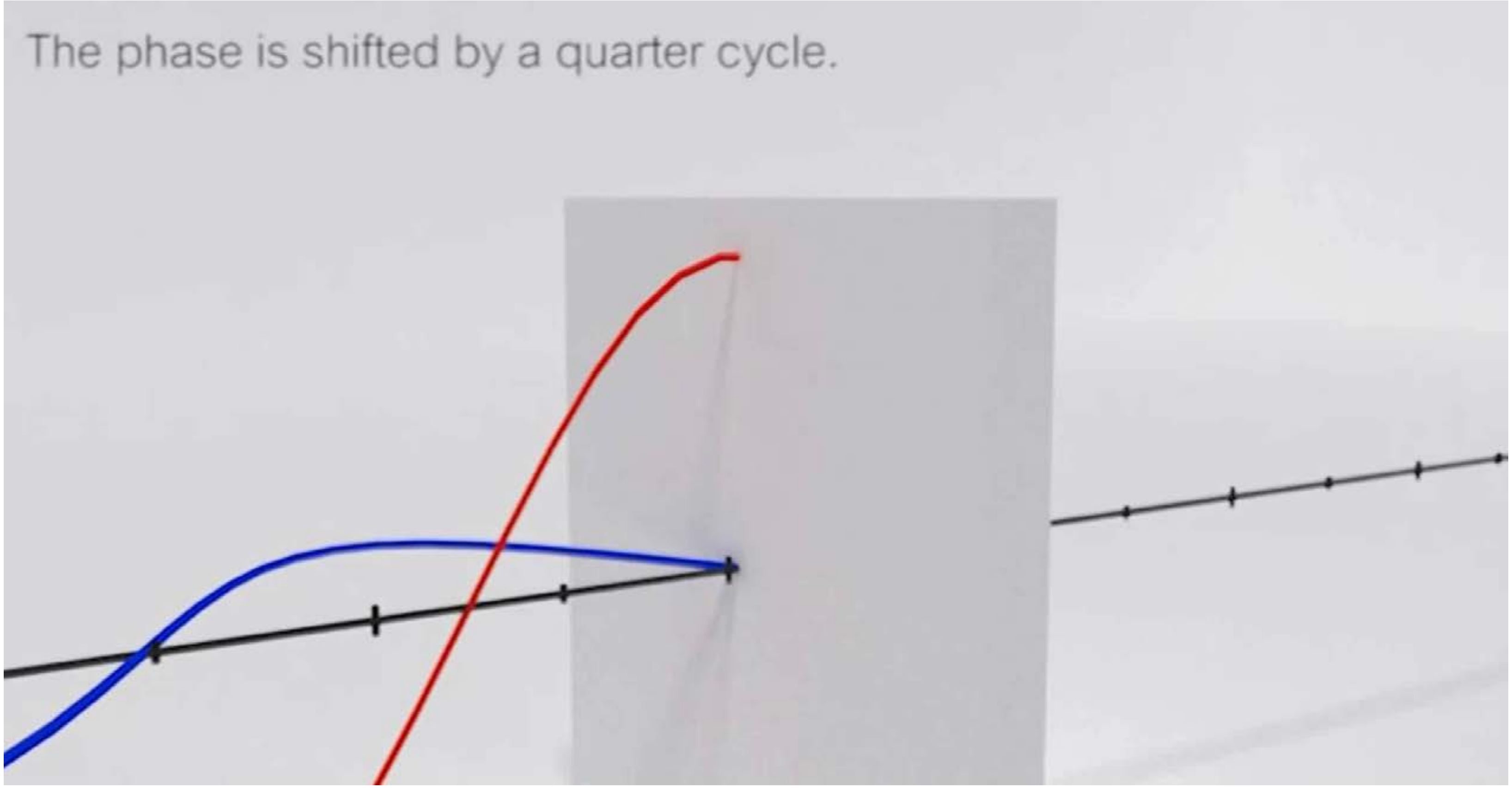
Note:
-Protons are positive
-Electrons are negative
-Neutrons are neutral

Select Mechanism to Interact with the Selected Material

- Electromagnetic fields are a primary source for transferring energy and angular momentum to electrons in the ^{40}Ca atom via electromagnetic force

- Bound state electrons in an atom will absorb and emit discrete quantities of energy and units of angular momentum determined by
  - Difference between the two bound state energy levels
  - The initial and final total angular momentum (combination of both the electron’s orbital angular momentum and an “internal” angular momentum called “spin”)

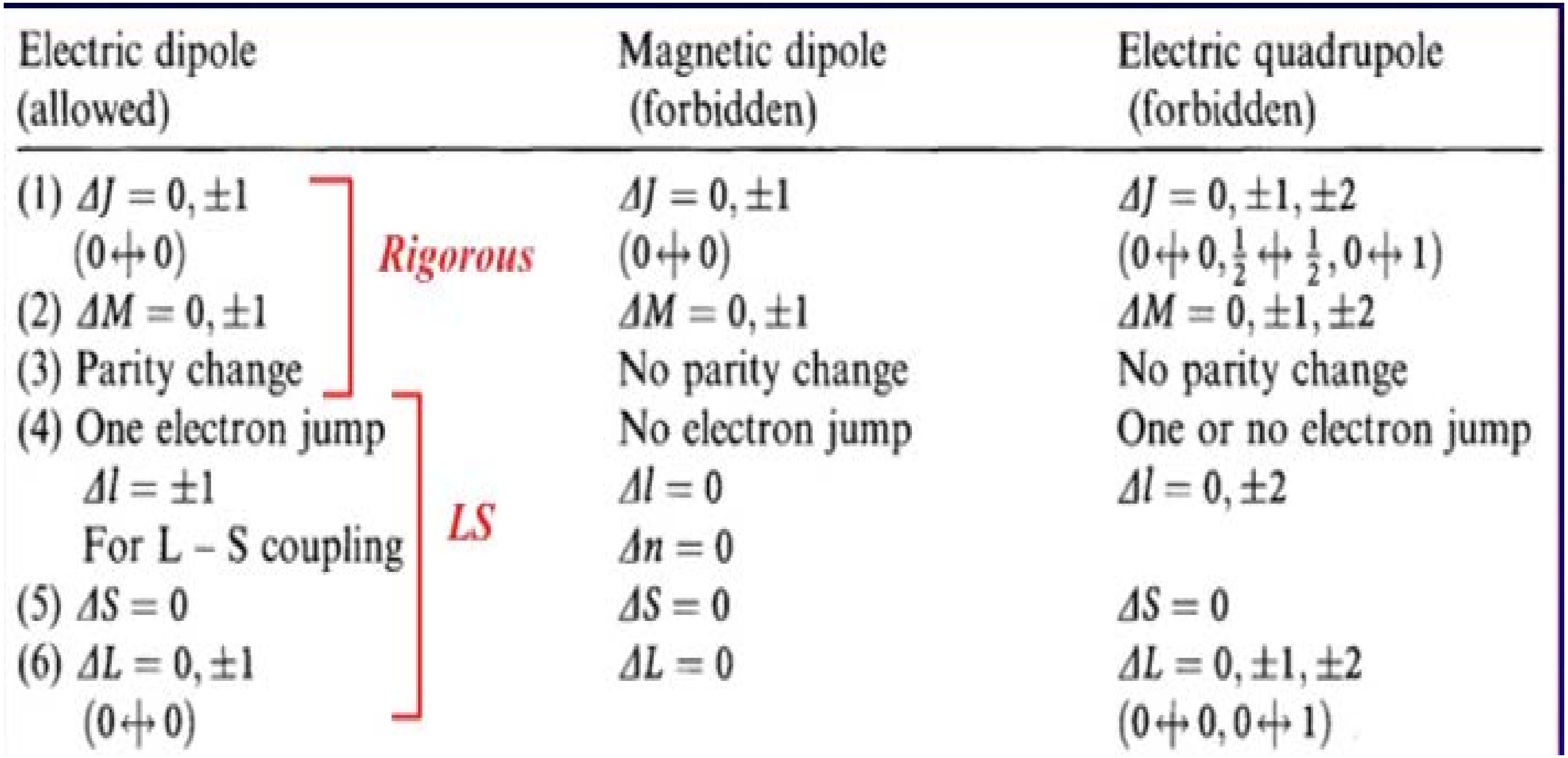
Propagation of Electromagnetic Fields



Transferring Energy to/from Bound State Electrons in a Material

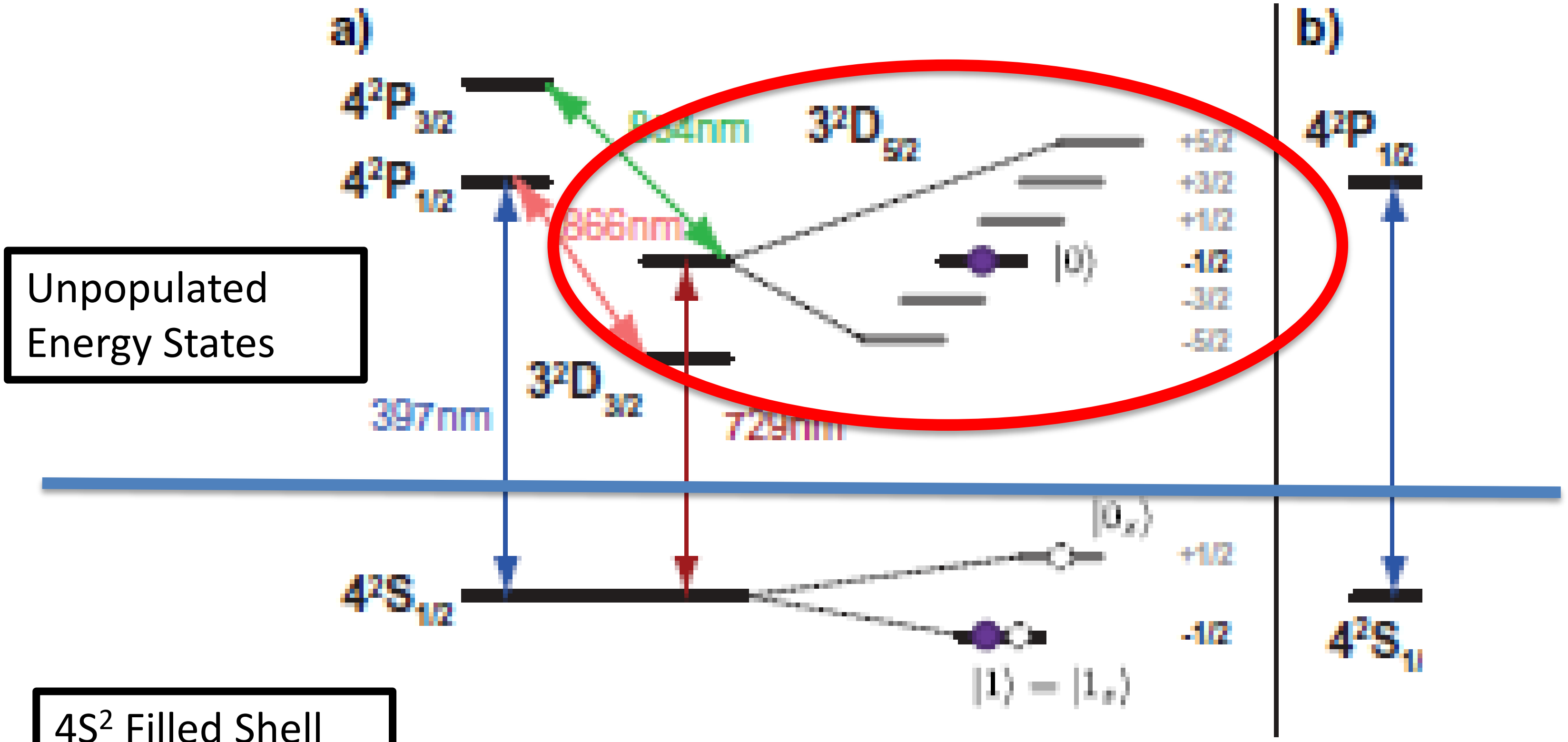
- By selecting a specific wavelength of electromagnetic radiation the experimentalist can control the
  - Energy absorbed or emitted by the electron
  - Discrete units of angular momentum transferred
- There are specific "quantum mechanics" rules constraining transitions between energy levels based on the transition energy and change in angular momentum (Selection Rules)

FYI – Background Design Information
QM Selection Rules for Atomic Spectra



Electric dipole (allowed)		Magnetic dipole (forbidden)	Electric quadrupole (forbidden)
(1) $\Delta J = 0, \pm 1$	*Rigorous*	$\Delta J = 0, \pm 1$	$\Delta J = 0, \pm 1, \pm 2$
$(0 \not\leftrightarrow 0)$		$(0 \not\leftrightarrow 0)$	$(0 \not\leftrightarrow 0, \frac{1}{2} \not\leftrightarrow \frac{1}{2}, 0 \not\leftrightarrow 1)$
(2) $\Delta M = 0, \pm 1$		$\Delta M = 0, \pm 1$	$\Delta M = 0, \pm 1, \pm 2$
(3) Parity change		No parity change	No parity change
(4) One electron jump	*LS*	No electron jump	One or no electron jump
$\Delta l = \pm 1$		$\Delta l = 0$	$\Delta l = 0, \pm 2$
For L – S coupling		$\Delta n = 0$	
(5) $\Delta S = 0$		$\Delta S = 0$	$\Delta S = 0$
(6) $\Delta L = 0, \pm 1$		$\Delta L = 0$	$\Delta L = 0, \pm 1, \pm 2$
$(0 \not\leftrightarrow 0)$			$(0 \not\leftrightarrow 0, 0 \not\leftrightarrow 1)$

Focus on the Atomic Spectra of $^{40}_{20}\mathrm{Ca}$



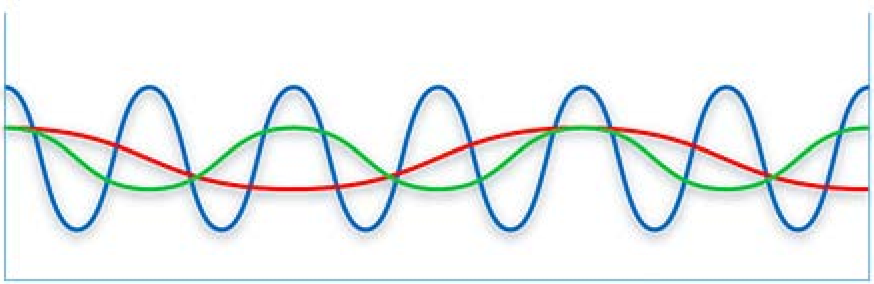
Unpopulated
Energy States

$4S^2$ Filled Shell
Energy States

Lasers

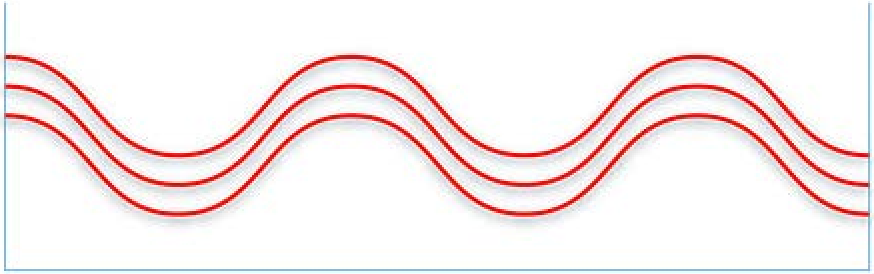
Electromagnetic Radiation Properties

- Light is composed of many electromagnetic fields of many different energies (frequencies)



Incoherent Light

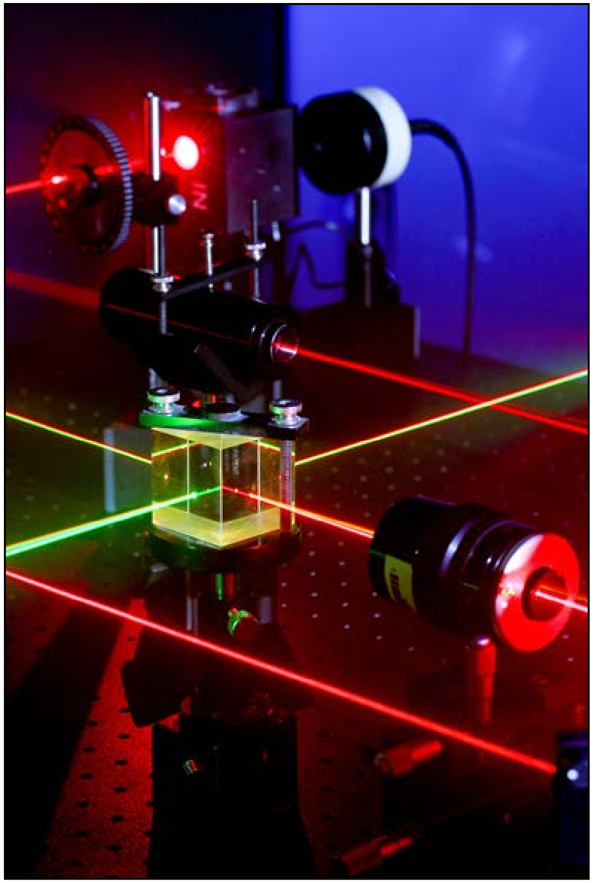
- Need light with properties of coherence (light with specific frequency and common phase)



Coherent Light

Need a Focused Source of Energy - Lasers -

- Lasers (coherent light source) allow experimentalists to “dial-up” a specific wavelength that will cause the electron to transition (resonate) between two different energy levels



Lasers in the Experimental Apparatus

- By varying the laser’s
  - Polarization
  - Wavelength
  - Duration of the laser light pulse

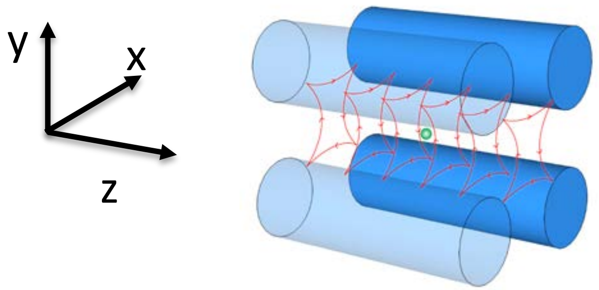
  the behavior of the electron can be controlled

- From a quantum computing perspective this is an effective mechanism for creating rotations and transformations

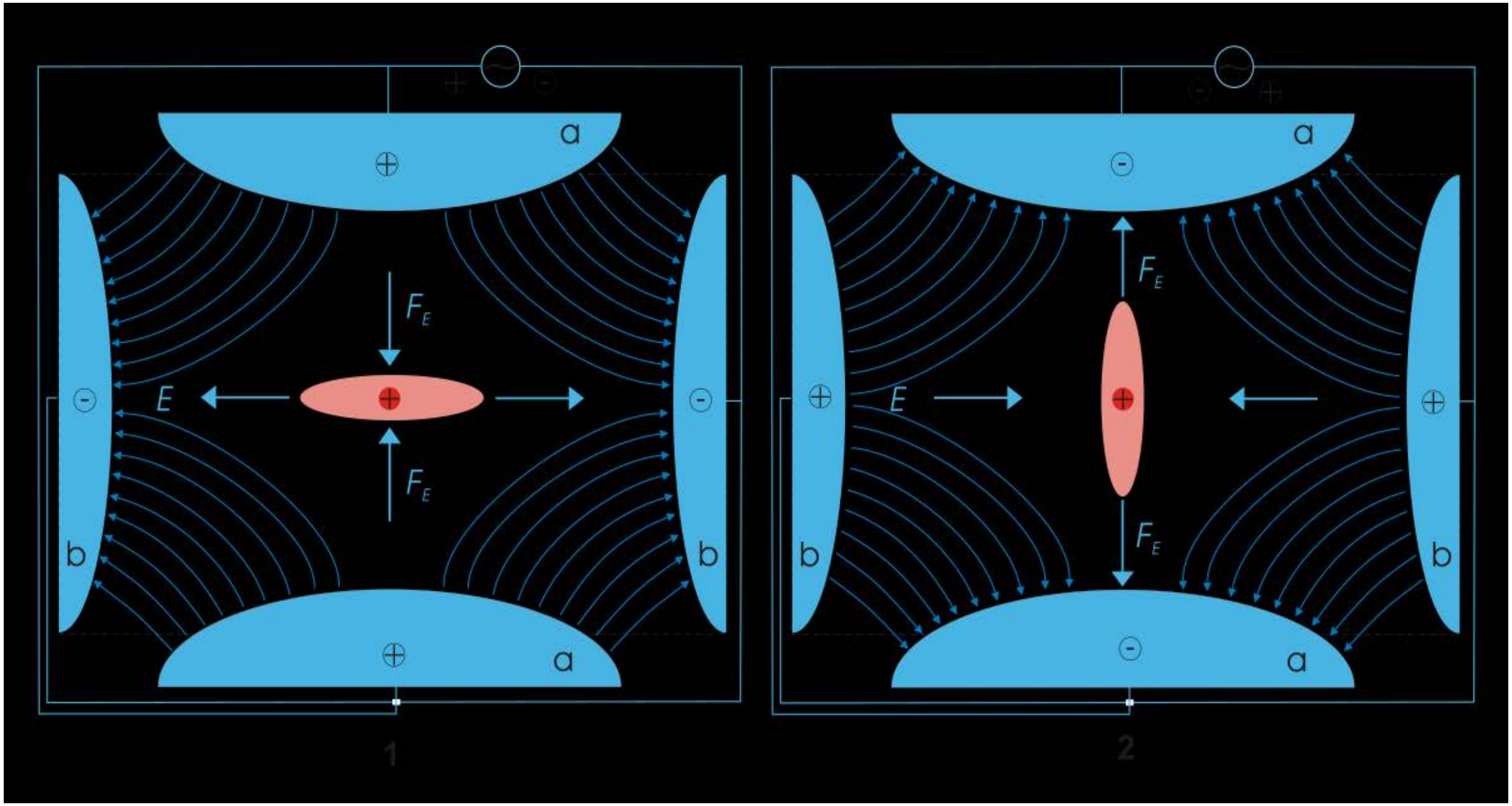
Construct the
TIQC Experimental Apparatus

Confine the Atoms into a “Device” Carl Friedrich Gauss’s Objection

- Static electric field confinement of the atoms in three dimensions is not possible
- Div **E** = 0 → no net inward force to constrain motion of the atoms
- Force cannot be inward in all directions → at least one direction where ions can escape

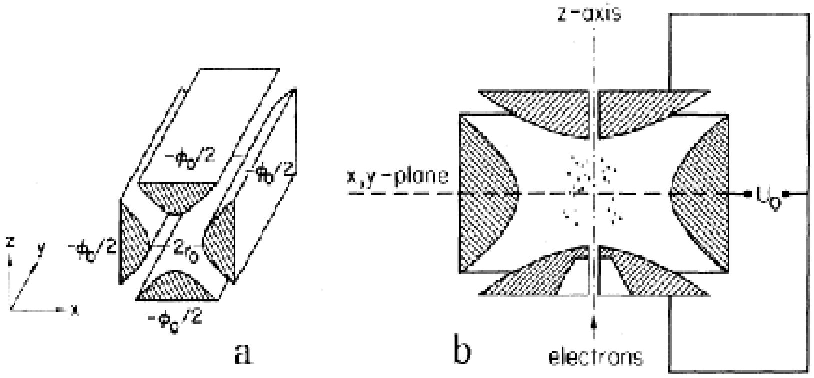


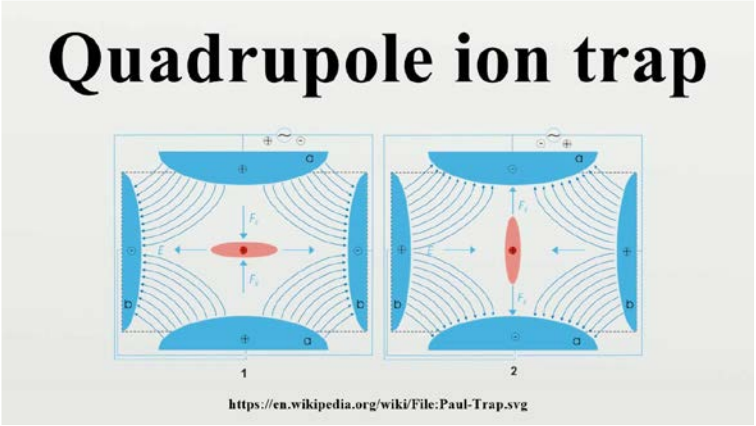
Consider Quadrapole Field



Put the Ca Atoms into “Trap” Apparatus

- Construct an apparatus that will confine ions along one dimension
- Consider a static quadrapole field





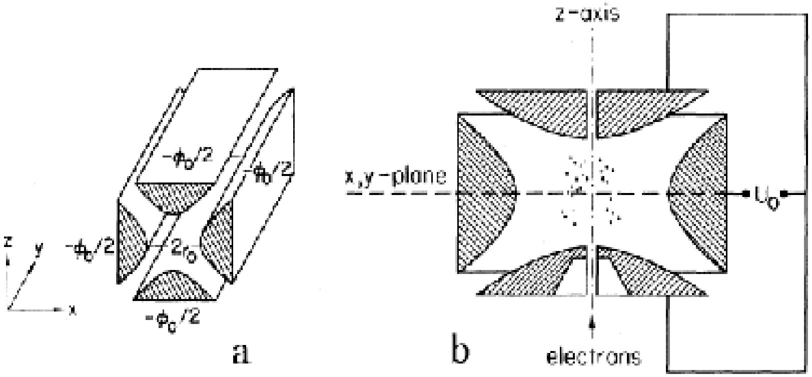
https://en.wikipedia.org/wiki/File:Paul-Trap.svg

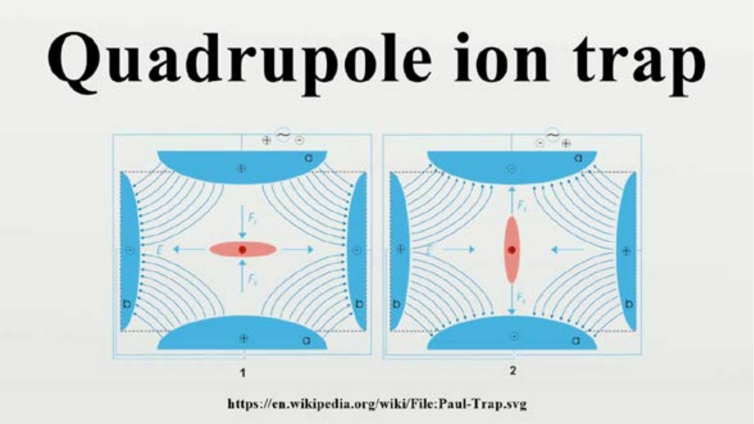
Model an Ion in a Stationary Quadrupole Field*

* You tube video (stationary saddle) https://www.youtube.com/watch?v=XTJznUkAmIY

Put the Ca Atoms into “Trap” Apparatus

- Modify the stationary quadrapole field
- Make a periodic rotation of the shape of the field lines as seen by the ion by applying an RF voltage
- In addition, the ends of the cylinders are biased at different dc voltages from the cylinder center so that the charged ions are axially confined





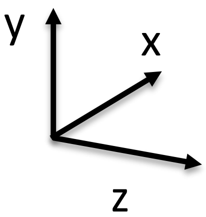
https://en.wikipedia.org/wiki/File:Paul-Trap.svg

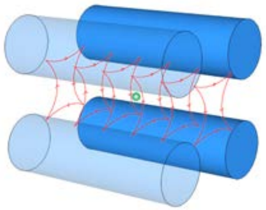
Rotating Saddle Point Surface*

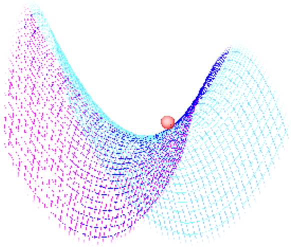
* You tube video https://www.youtube.com/watch?v=rJ13qwRYs

Additional Effect of the Periodic RF Potential

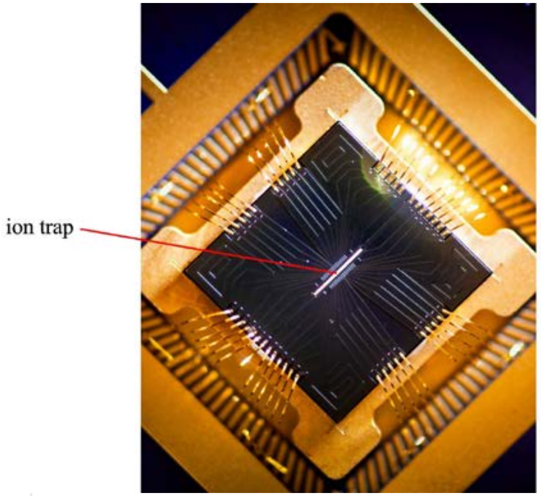
- Net effect produces a combined effect
  1. The combination of the RF and DC voltages also produce a harmonic potential
  2. The electrostatic repulsion of each ion creates a string of ions trapped along the z-axis of the trap
  3. Under these conditions the motion of the confined ions becomes quantized as a 1-dimensional harmonic oscillator with equally spaced energy levels $\hbar\omega$

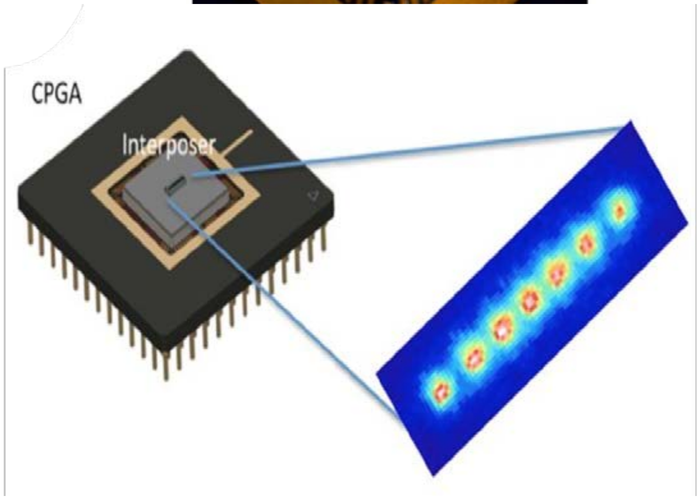






Ions Implanted within Trap on a Chip

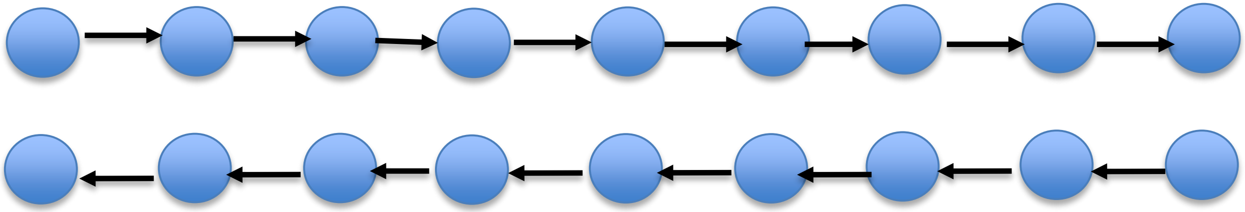




K. R. Brown, J. Kim, C, Monroe, Co-designing a scalable quantum computer with trapped atomic ions, Quantum Information **2**, 16034 (2016).

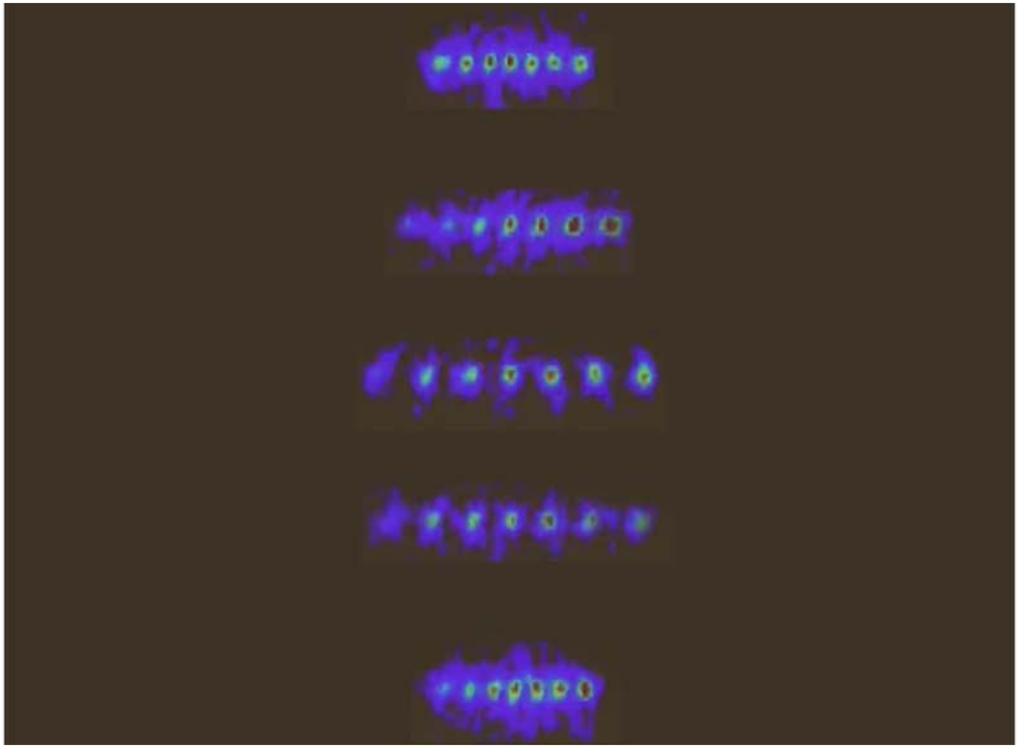
Trapped Ion Vibrational Modes (Phonons)

- These ions are stored in the trap at room temperature
- The ions have many thermal vibrational modes (phonons)



- Phonons in this context are center of mass energy eigenstates that represent the coupled vibrational modes of the entire lattice of ions

Phonons Have Many Vibrational Modes*

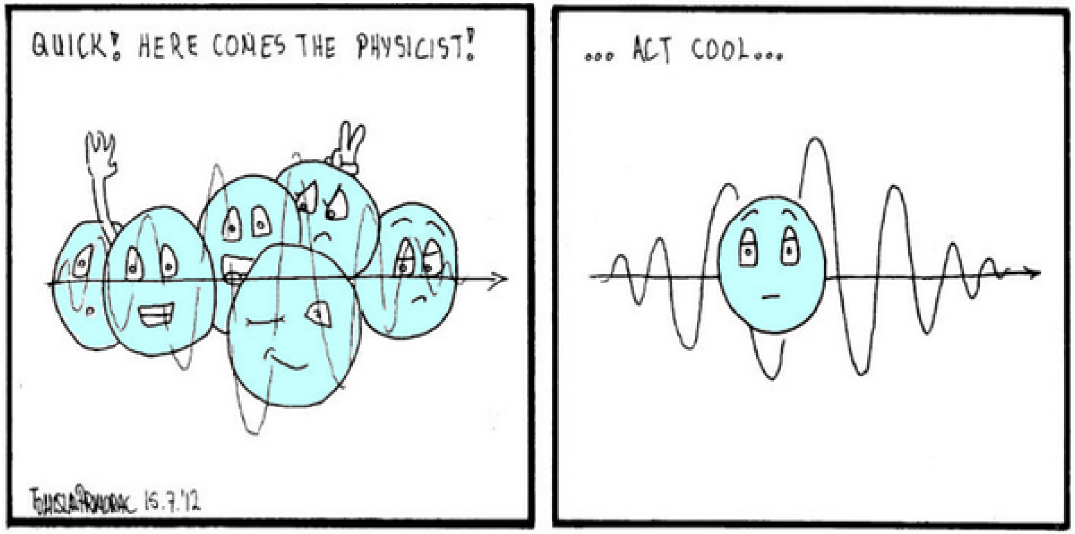


* Ion Trapping, C. Monroe, 12th Canadian Summer School on Quantum information, University of Waterloo, 2012

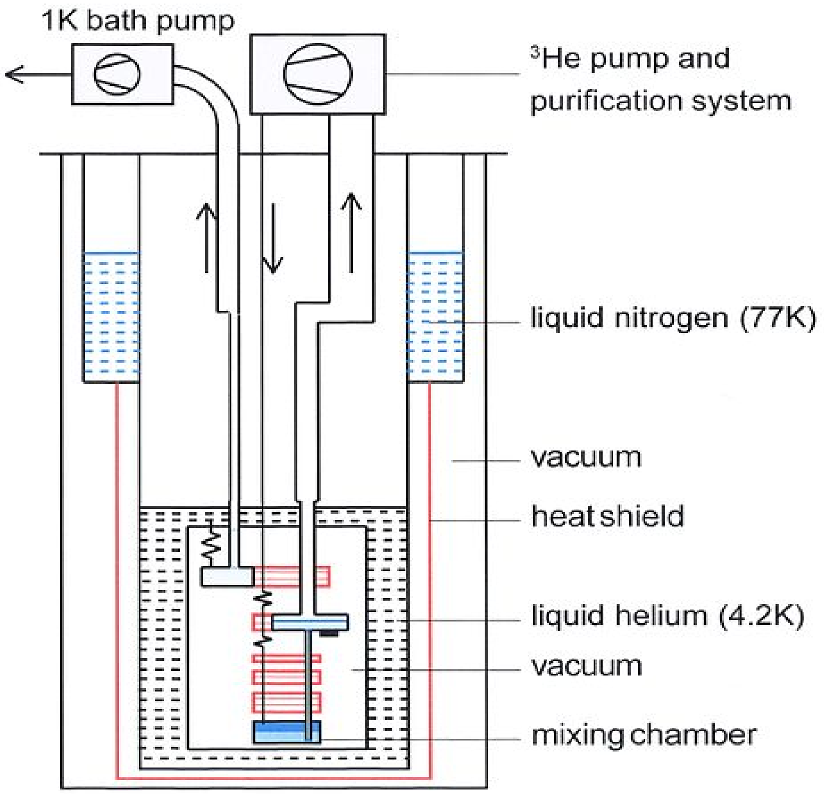
“Low Temperature” Requirement for the TIQC Apparatus

Low Temperature Requirement for the Experimental Apparatus

- Electrons in ions and the ions themselves are subject to many types of energy fluctuations at room temperature
- There are many excited states to which the electron and the ion can transition (unwanted volunteers)
- Suppress this “jitter” by cooling the material



Recall The IBM Superconducting Transmon Design Used Dilution Refrigerators *



* Image from http://www.wikiwand.com/en/Dilution_refrigerator

Low Temperature Experimental Apparatus IBM Q Quantum Computer Cryostat

- **TIQC Apparatus Usually Operates at Room Temperature**

- **Uses Different Physics Principles From IBM QC Hardware Platforms to Cool the Ions**

Construct a “Low Temperature” Environment on the ^{40}Ca Electrons and Ions

Goals

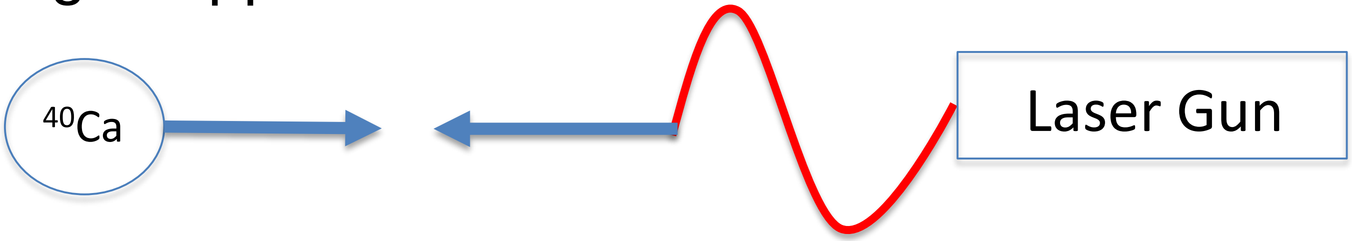
1. Want only a few excited states above the ground state are accessible to the ion (this limits the size of the Hilbert space available for energy transitions)
2. The trap must form a 1D harmonic oscillator potential that stores the ^{40}Ca ions
3. The ^{40}Ca ions should only exhibit lowest level vibrational states (phonons) in the 1 dimensional harmonic oscillator potential when sufficiently cooled

Laser Cooling Design

- To manipulate the ions, they need to remain as stationary as experimentally feasible within the trap
- For a TIQC platform construct a ultrahigh vacuum apparatus (< 10^{-6} atm pressure)
- In the ultra high vacuum lower the kinetic energy of the ion to as close to ground state as possible
- Use the laser and physics principle of energy exchange in collision to extract kinetic energy from the ions
- Result is that the ions will float insider room temperature vacuum chamber at temperature close to absolute zero

Doppler Cooling

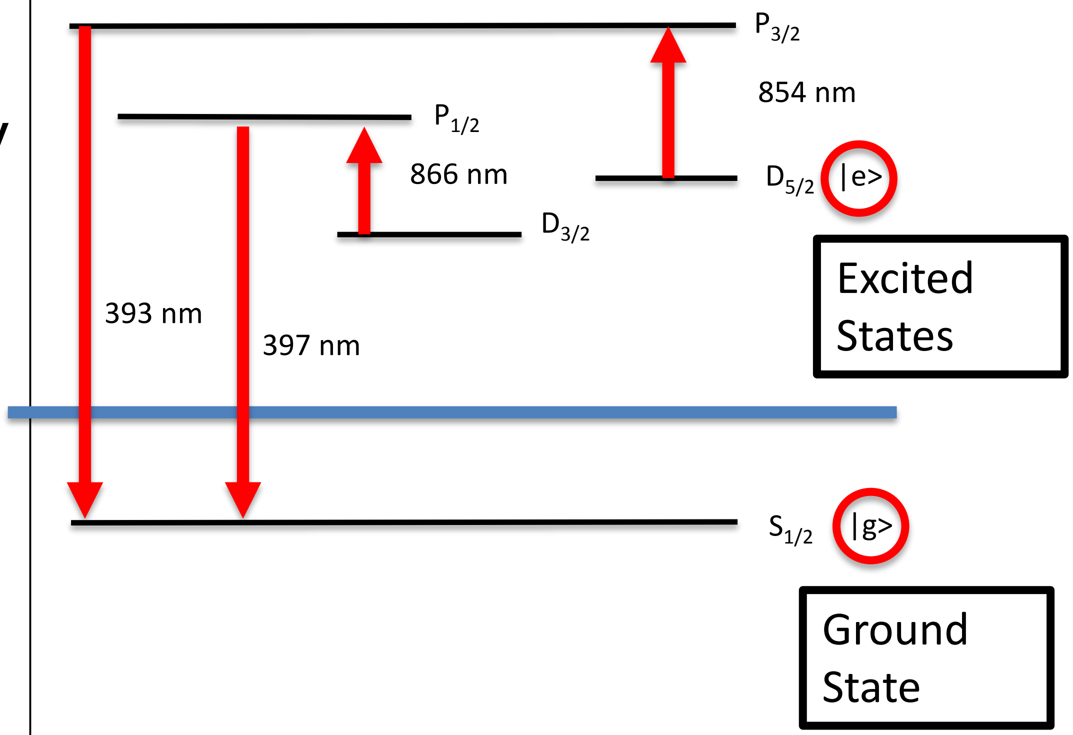
- A laser beam carries momentum in the photons that can "collide" with a Calcium Ion in the trap and decrease the ions kinetic energy if the ion and laser beam are travelling in opposite directions



- The ion will slow down when the photon from the laser beam is absorbed by the ion
- The ion will re-radiate the photon energy randomly in all directions keeping a net zero momentum change for the ion but decreasing the ion's kinetic energy
- Drops ion temperature to ~ 0.5 mKelvin

Manipulating the Ground and Excited States of the Electrons in the $^{40}_{20}$Ca atom

- Doppler cooling is applied to the electric dipole transition (397 nm)
- Small probability of decay to a 3$D_{3/2}$ requiring a 2nd laser to flush photons trapped in this stat
- Requires multiple lasers tuned to specific wavelengths to depopulate the unwanted excited states



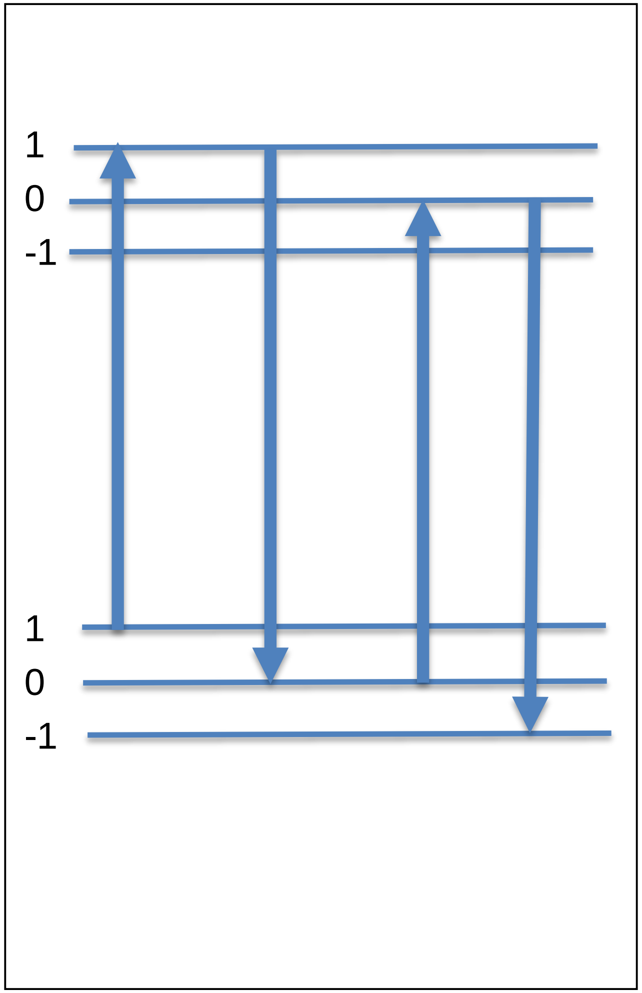
Excited States

Ground State

Optical Pumping to Lower Energy

- Laser tuned to differences in energy levels between hyperfine excited states and ground states
- When electron hits lowest hyperfine ground state there is no laser pulse with the exact energy difference for it to transition to a higher state
- In lowest energy state

Hyperfine State Transition Example



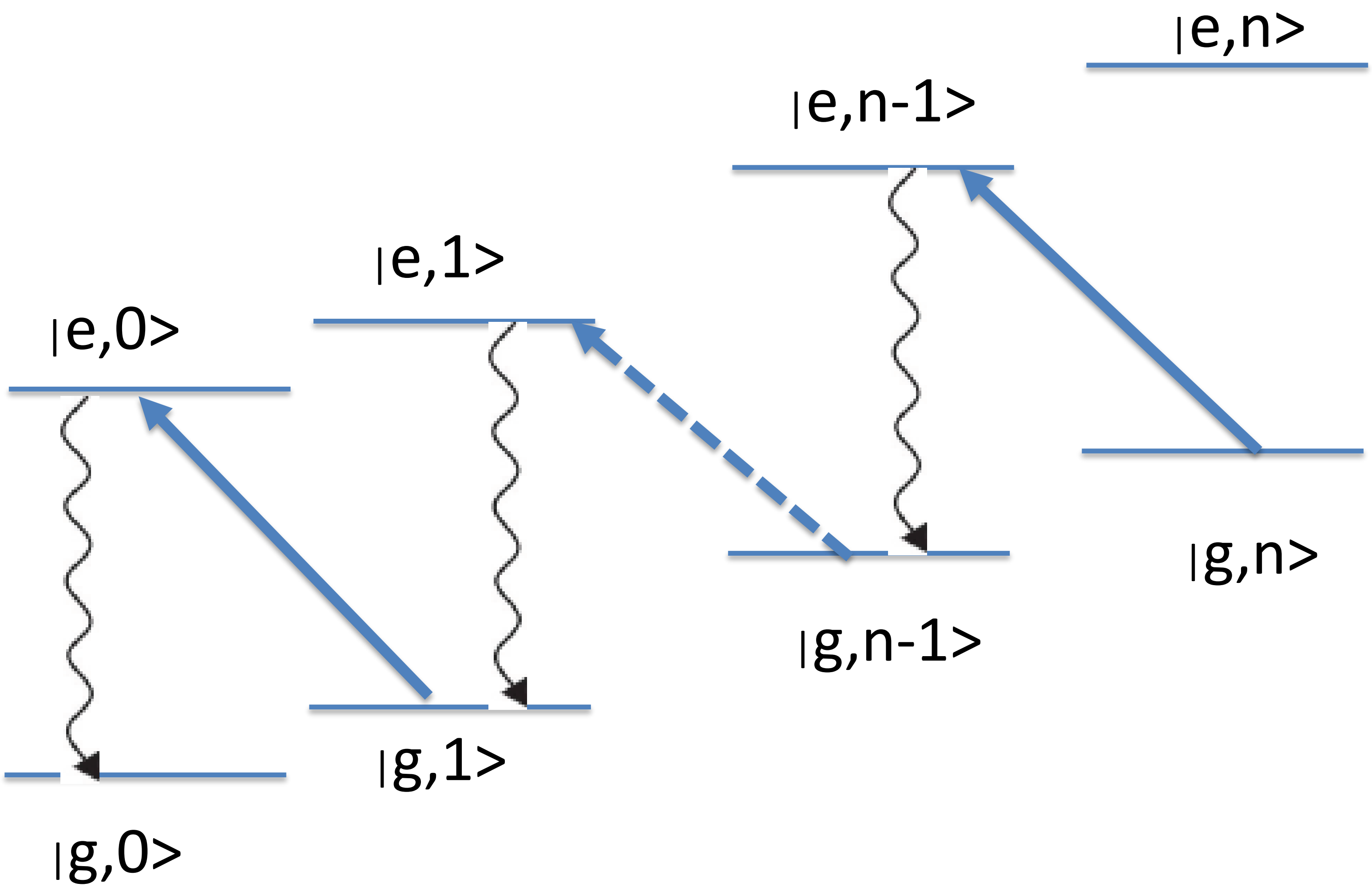
Sideband Cooling

- Trapped ions may be in different vibrational modes
- Tune sequence of lasers to be resonant with transition

|g,n> → |e,n-1> → |g,n-1>→… |g,1> → |e,0> → |g,0>

- Corresponding spontaneous emission will noave no change in vibration quantum number on average
- Cascading step-down emissions of energy until reach the |g,0> level which is the lowest energy state

Sideband Cooling



Summary of TIQC Device Properties

- Have a design for a 2 level spin system interacting with an electromagnetic field
- Spin is physically confined within a 1-dimensional harmonic oscillator potential
- Spin interactions controlled by rotations in response to a laser pulse
- States are quantized with energy of scale hν
- These harmonic oscillator bound states are identified as center of mass phonon vibrations
- Laser cooling quiets both the Ca electronic transitions and phonons excitations to lowest modes

Trapped Ion Quantum Computer Design

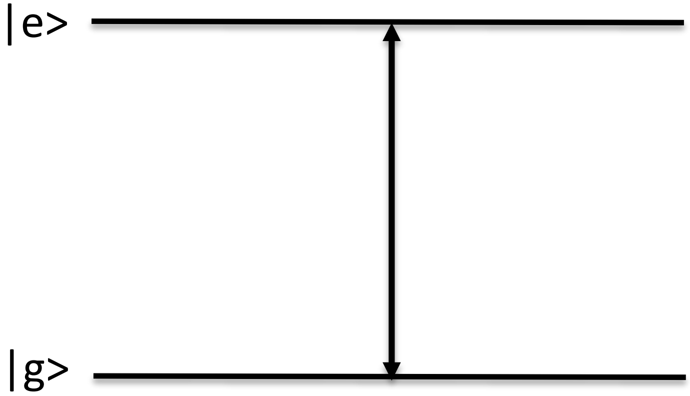
Simple Model of a Two Qubit Quantum Computer

- Construct a 2 level spin system interacting with an electromagnetic field
- Spin interactions controlled by a spin rotation in response to a laser
- Spin is physically confined within a harmonic potential
- States are quantized with energy of scale hν
- These harmonic oscillator bound states are identified as center of mass phonon vibrations

Outer Product Representation of Available Qubit Quantum States in a TIQC

Two level Ion

Harmonic Potential Trap



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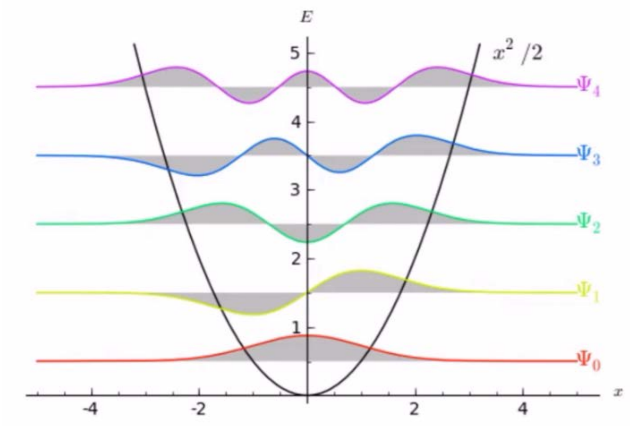
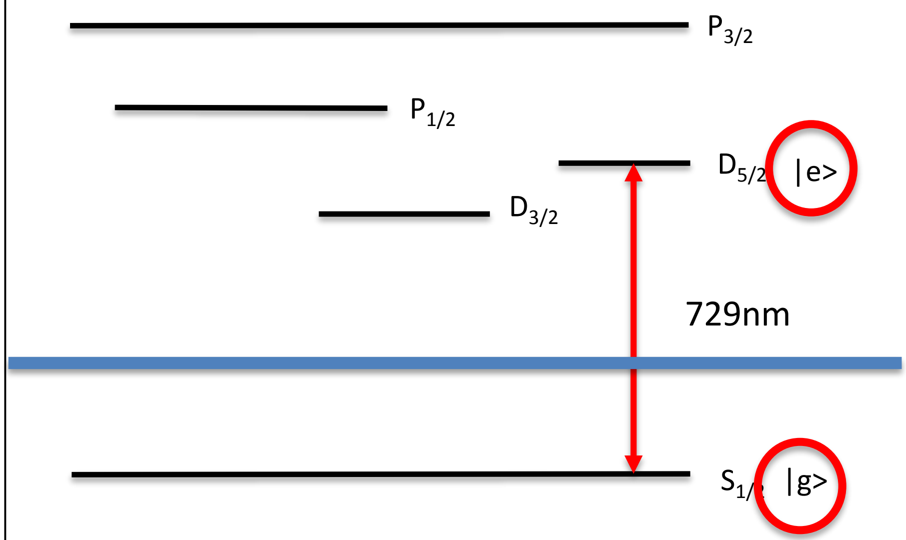


Building a 2 Level Ion Qubit State

- Want to identify an excited state that will be "long-lived"
- From laws of Quantum Mechanics ($S_{½} \rightarrow D_{5/2}$) is a "forbidden transition" and so the excited state will be long lived (~1 sec) compared to the lifetime of an allowed transition (~ 1 nanosecond)
- This transition can be identified as a potential candidate for a stable qubit



Ion Spin States

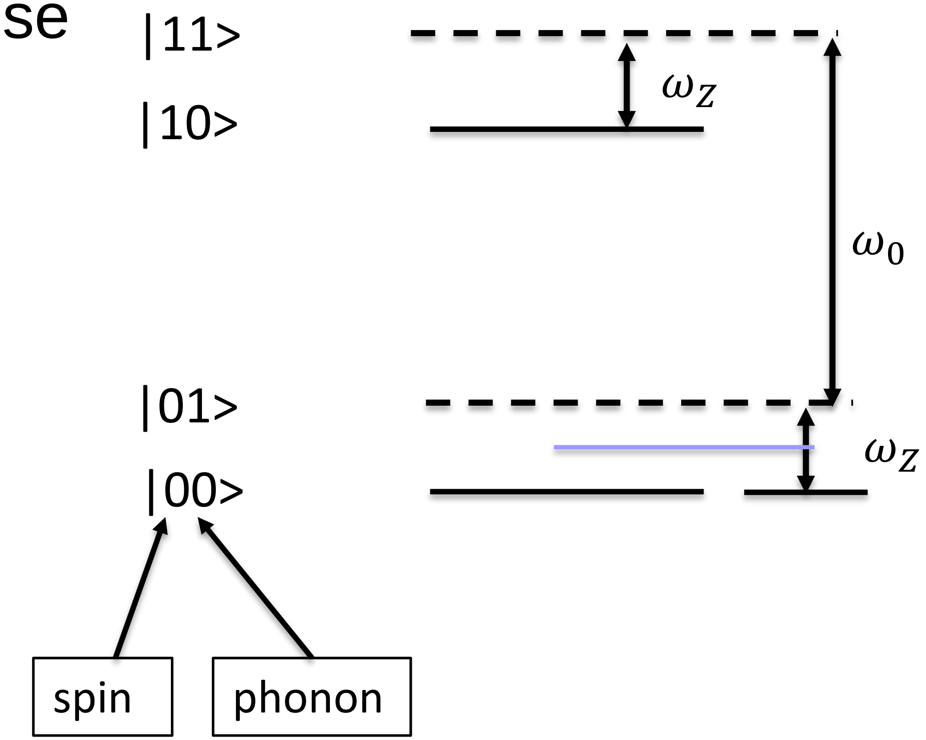
- Choose electromagnetic field (laser pulse) of appropriate frequency and duration
- Use the rotation operator to view the pulse as a unitary transformation allowing a one qubit transformation to be performed on the spin state

$$R_x(\theta) = \exp(-i\theta S_x)$$

$$R_y(\theta) = \exp\left(-i\theta S_y\right)$$

Phonon Vibration States

- Assume particle is cooled so that it is near its lowest vibrational state
- Have a ladder of these harmonic oscillator states



Step 2.

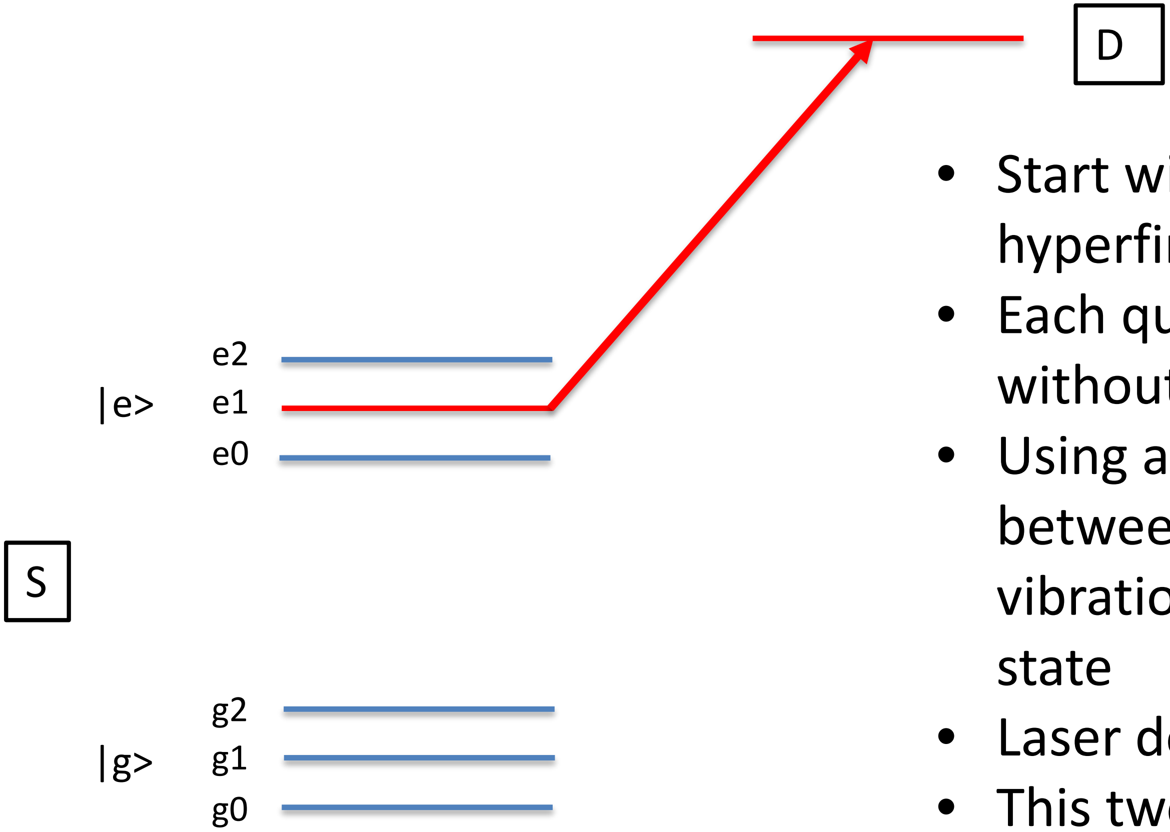
Construct Universal Quantum Gates Without Collapsing The Entire Quantum Computing Computation

Identify Candidates for a Two Qubit System

- Identify a single 2 level spin system interacting with an electromagnetic field
  a) Qubit can be identified by the quantized bound states of an atomic material as seen through ability of a spin to respond to an electromagnetic field
  b) A second qubit can be identified through the set of interactions of the ion's vibrational modes

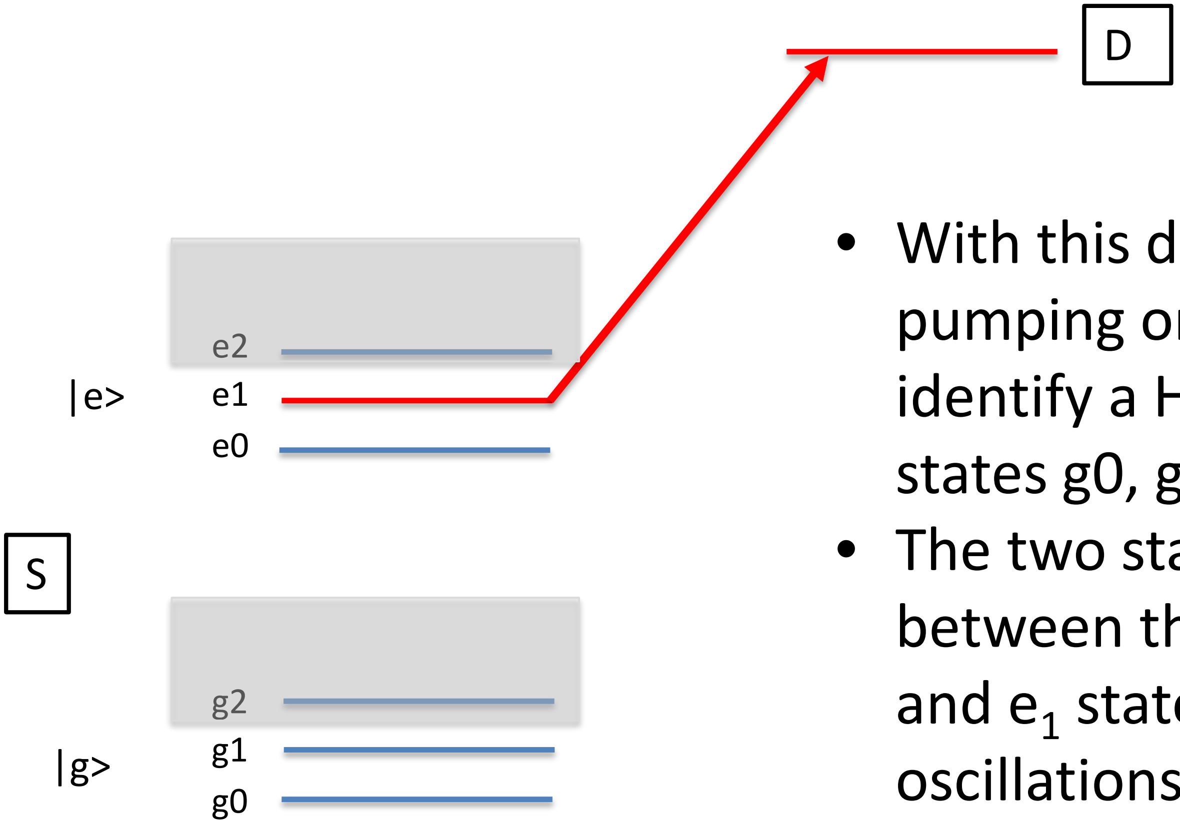
ν

Single Ion Behavior



- Start with ion in an S state with 2 hyperfine states
- Each qubit has |g> and |e> without center of mass motion
- Using a laser select resonance between the |e1> excited vibrational state of |e> to a D state
- Laser does not affect g_0, g_1 or e_1
- This two state laser driven pulse produces Rabi oscillations

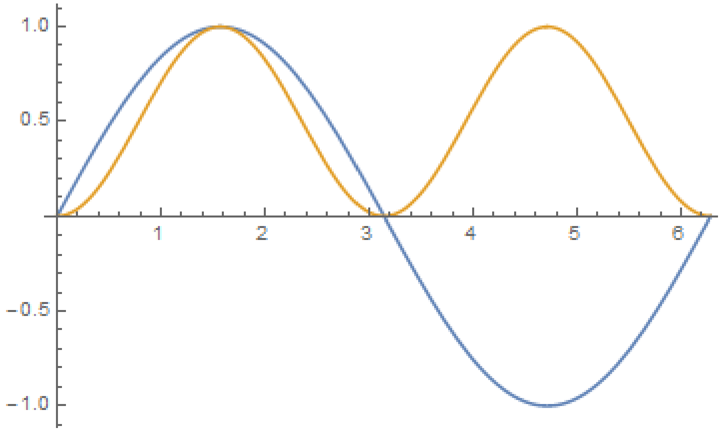
Construct a Phase Gate



- With this driven laser pulse pumping only this transition identify a Hilbert space with states g0, g1, e0, e1
- The two state oscillation between the auxiliary D state and e_1 state produce Rabi oscillations

Rabi Oscillations

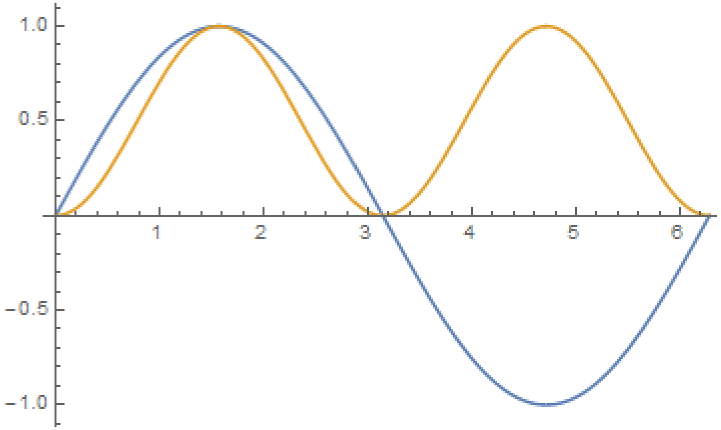
- Rabi oscillations (also known as the Rabi cycle or Rabi flop) is the cyclic behavior of a two-level quantum system in the presence of an oscillatory driving field (such as a laser pulse)
- Figure below shows cyclic probability amplitude (blue) and the measurement probability (yellow)



Rabi Oscillations Information Used to Create a Phase Gate

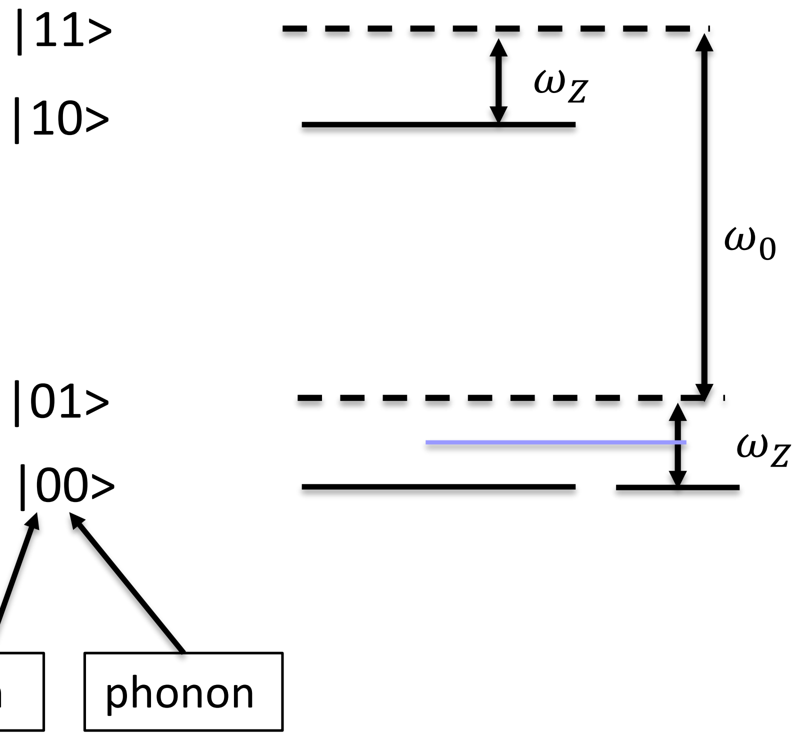
- Rabi oscillation after one period has changed the phase of the probability amplitude by π (phase is -*i*) (blue)
- Quantities measured in the lab are the probabilities (yellow)
- Figure shows that after the system has returned to the original state the probability has shifted by 2π but the phase by π (-*i*)
- 2π pulse in population shifts phase of wavefunction by π (-*i*)

$g_0 \rightarrow g_0$
$g_1 \rightarrow g_1$
$e_0 \rightarrow e_0$
$e_1 \rightarrow -e_1$



Phonon Vibration States for Single Atom

- Assume particle is cooled so that it is near its lowest vibrational state
- Have a ladder of these harmonic oscillator states



|11>

|10>

|01>

|00>

ω_Z

ω_0

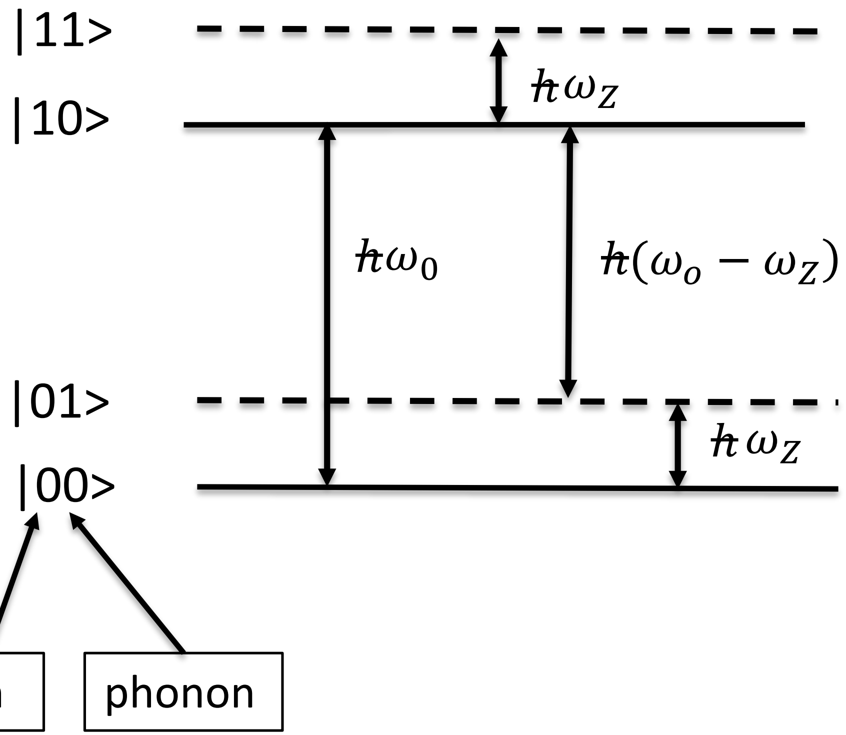
ω_Z

spin

phonon

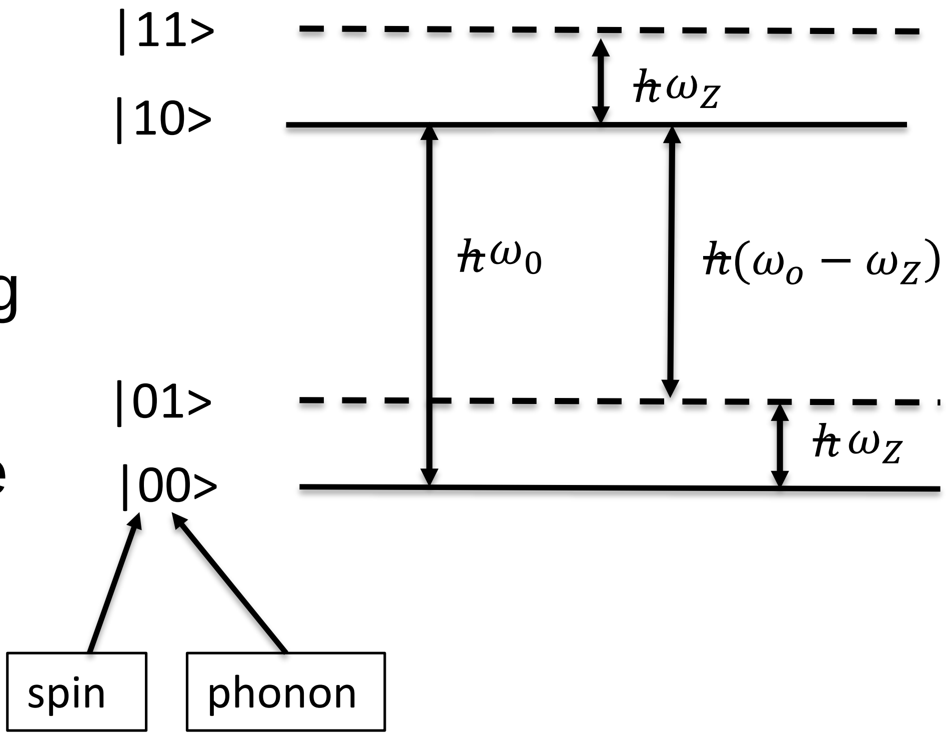
Energy Levels of a Single Atom

- Assume ion is cooled so that it is near its lowest vibrational state
- Have a ground and excited spin state and a ground and excited vibrational phonon state



Energy Levels of a Single Atom

- Use a laser detuned from the $\hbar\omega_0$ spin transition tuned to |10>→|01> transition energy $\hbar(\omega_o - \omega_Z)$
- Uniquely forces a transition from |10> → |01> without possibility of inducing other transitions
- This places the entire ion chain in the first excited vibrational state of spin |0>



Construct QM Basis State for Two ^{40}Ca Atoms

- Construct a set of basis vectors from a linear vector space describing wavefunction of two ions (A and B) and a collective phonon vibrational state

$$|0_A > |0_B > |0 >$$

$$|0_A > |1_B > |0 >$$

$$|1_A > |0_B > |0 >$$

$$|1_A > |1_B > |0 >$$

1. Laser Pulse Generates a π Rotation Pulse Directed to Ion A

- Select two ^{40}Ca ions (A and B) and the collective phonon state of the chain of ^{40}Ca ions and construct outer product state
- Construct operator U_A that describes a π pulse directed to ion A with energy $\hbar(\omega_o - \omega_Z)$
- The laser pulse generates Rabi oscillations
- Ion A generates phase –*i, changes ion A from |1>→|0> and phonon vibrational state |0>→|1> (ion B unaffected)*

$$
\begin{aligned}
|0_A>|0_B>|0> &\longrightarrow |0_A>|0_B>|0> \\
|0_A>|1_B>|0> &\longrightarrow |0_A>|1_B>|0> \\
|1_A>|0_B>|0> &\longrightarrow -i\,|0_A>|0_B>|1> \\
|1_A>|1_B>|0> &\longrightarrow -i\,|0_A>|1_B>|1>
\end{aligned}
$$

2. Generate Laser Pulse Directed to Ion B

- Construct operator V_B that generates a π pulse directed to ion B and changes the phase of the wavefunction by π
- Occurs only if ion B is the ground state |0> and the phonons are in excited vibrational state |1>

$$
\begin{aligned}
|0_A>|0_B>|0> &\longrightarrow |0_A>|0_B>|0> \\
|0_A>|1_B>|0> &\longrightarrow |0_A>|1_B>|0> \\
\boxed{-i}\,|0_A>\boxed{|0_B>}\boxed{|1>} &\longrightarrow \boxed{i}\,|0_A>|0_B>|1> \\
-i\,|0_A>|1_B>|1> &\longrightarrow -i\,|0_A>|1_B>|1>
\end{aligned}
$$

3. Apply Operator U_A a Second Time with a π Pulse Directed to Ion A

- π pulse again directed to ion A
- If ion A is in state |0> and phonon is in |1> generates a phase rotation of –*i* and changes the state of ion A from |0>→|1> and the vibrational phonon state from |1>→|0>

$$\begin{array}{lcl}
|0_A>|0_B>|0> & \longrightarrow & |0_A>|0_B>|0> \\
|0_A>|1_B>|0> & \longrightarrow & |0_A>|1_B>|0> \\
i\,|0_A>|0_B>|1> & \longrightarrow & 1\,|1_A>|0_B>|0> \\
-i\,|0_A>|1_B>|1> & \longrightarrow & -|1_A>|1_B>|0>
\end{array}$$

Construct a 2 Qubit Truth Table for the Product Operation $W=U_AV_BU_A$

Initial State

$|0_A>|0_B>|0>$

$|0_A>|1_B>|0>$

$|1_A>|0_B>|0>$

$|1_A>|1_B>|0>$

$$W_{CPHASE} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Final State

$|0_A>|0_B>|0>$

$|0_A>|1_B>|0>$

$|1_A>|0_B>|0>$

$-|1_A>|1_B>|0>$

Recall The Property of a Control Phase Gate

- In a $\binom{1}{0}, \binom{0}{1}$ basis, the Control Phase gate changes the sign of the 2nd qubit when the 1st qubit is 1

$$W_{CPHASE} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

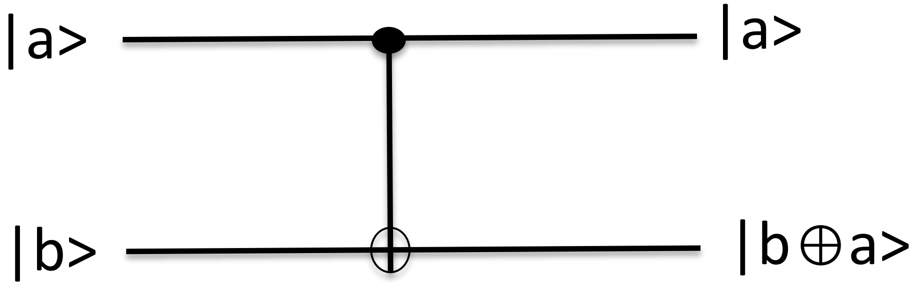
$$W_{CPHASE}^{\dagger} W_{CPHASE} = I$$

- The CPHASE gate becomes a CNOT universal quantum gate when combined with 2 Hadamard gates

Recall the Property of the CNOT Gate

Matrix representation of the CNOT gate

$$U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad U^{\dagger}_{CNOT} U_{CNOT} = I$$

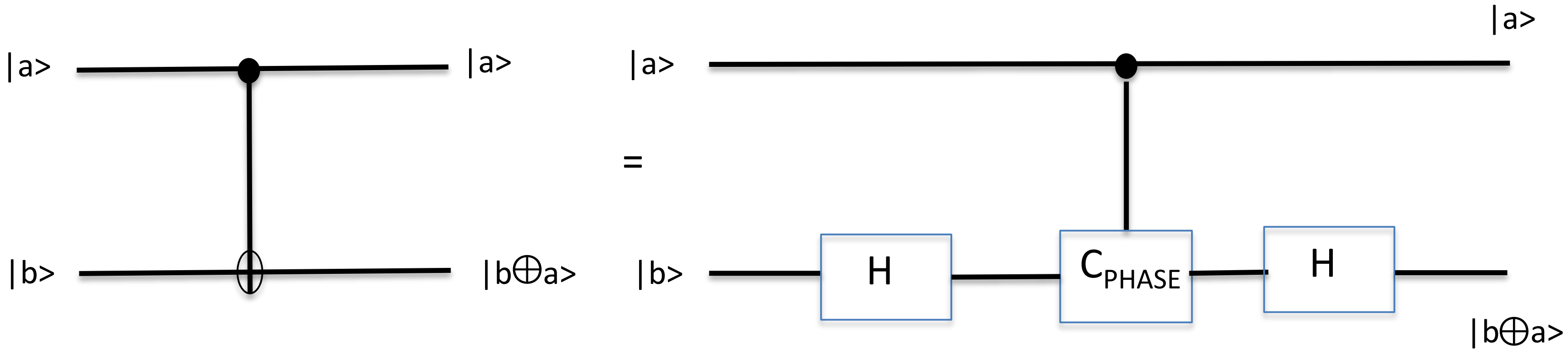


$|a> = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $|b> = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

|aa> → |aa>
|ab> → |ab>

|ba> → |bb>
|bb> → |ba>

Express CNOT in Terms of CPHASE



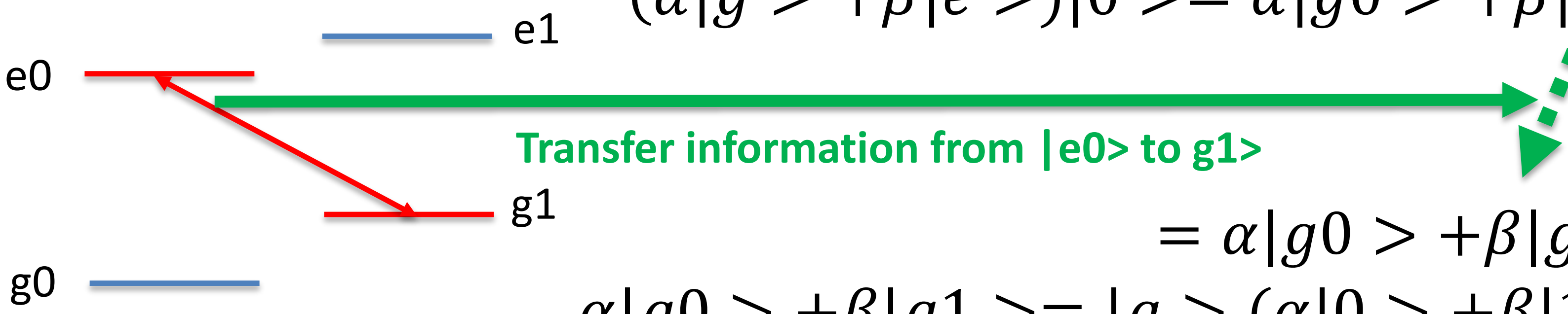
SUMMARY - Operation of the Trapped Ion Quantum Computer

1. Select any two ions ("A" and "B") in the 1-dim chain of ^{40}Ca
2. Generate a laser pulse to force ion "B" into an up spin state
3. If the ion B is spin up use another laser pulse to induce center of mass motion of the ion chain (common dipole motion)
4. The center of mass (CoM) motion is uniformly detected everywhere along the ion chain
5. Swap the information from the up state of ion B to the center of mass motion of the ion chain (essentially communicate signal on the "data bus" of ion chain that the ion "B" is spin up)

Information SWAP Between Ion Spin State and Phonon Center of Mass Vibrational State

- g_0 and e_0 are the internal states of the ion
- Construct arbitrary qubit state $(\alpha|g>+\beta|e>)$ with the center of mass motion |0> laser cooled to ground state
- Fire another π pulse this time between states |e0> and |g1>
- Probability amplitudes α and β transferred from the internal spin state of the ion to the phonon vibrational center of mass state

$$(\alpha|g>+\beta|e>)|0>=\alpha|g0>+\beta|e0>$$



Transfer information from |e0> to g1>

$$=\alpha|g0>+\beta|g1>$$

$$\alpha|g0>+\beta|g1>=|g>\underbrace{(\alpha|0>+\beta|1>)}$$

Information now in phonon state that is center of mass motion

Operation of the Trapped Ion Quantum Computer

6. Communicate Ion B information to Ion A by constructing a phase gate via the data bus (CoM motion of phonons)
7. Change rotation of the wavefunction but only if both ions are spin up
8. Replace the information on the data bus back into the original Ion B (this clears the data bus)
9. Now have a measurement of Ion A's state without disturbing it in a way that collapses the entire TIQC state wavefunction
10. Quantum computation can continue to next gate operation

NxN Qubit Communications

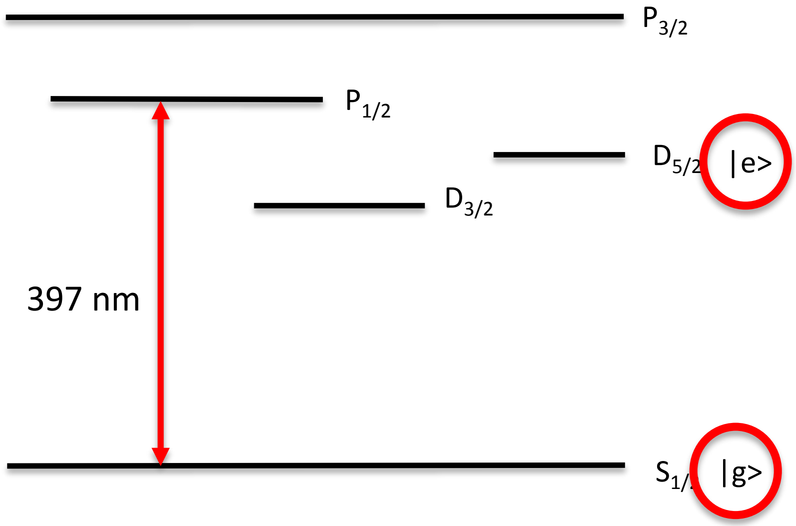
- These 2 ions form quantum computing 2 qubit operations
- Can operate a TIQC with many ions that provide 2^N states using combinations or any 2 ions remotely separated from each other
- This procedure selects only the 2 ions that participate in the interaction while all other ions in the chain are undisturbed (no measurement disturbance of the wavefunction)

Step 3

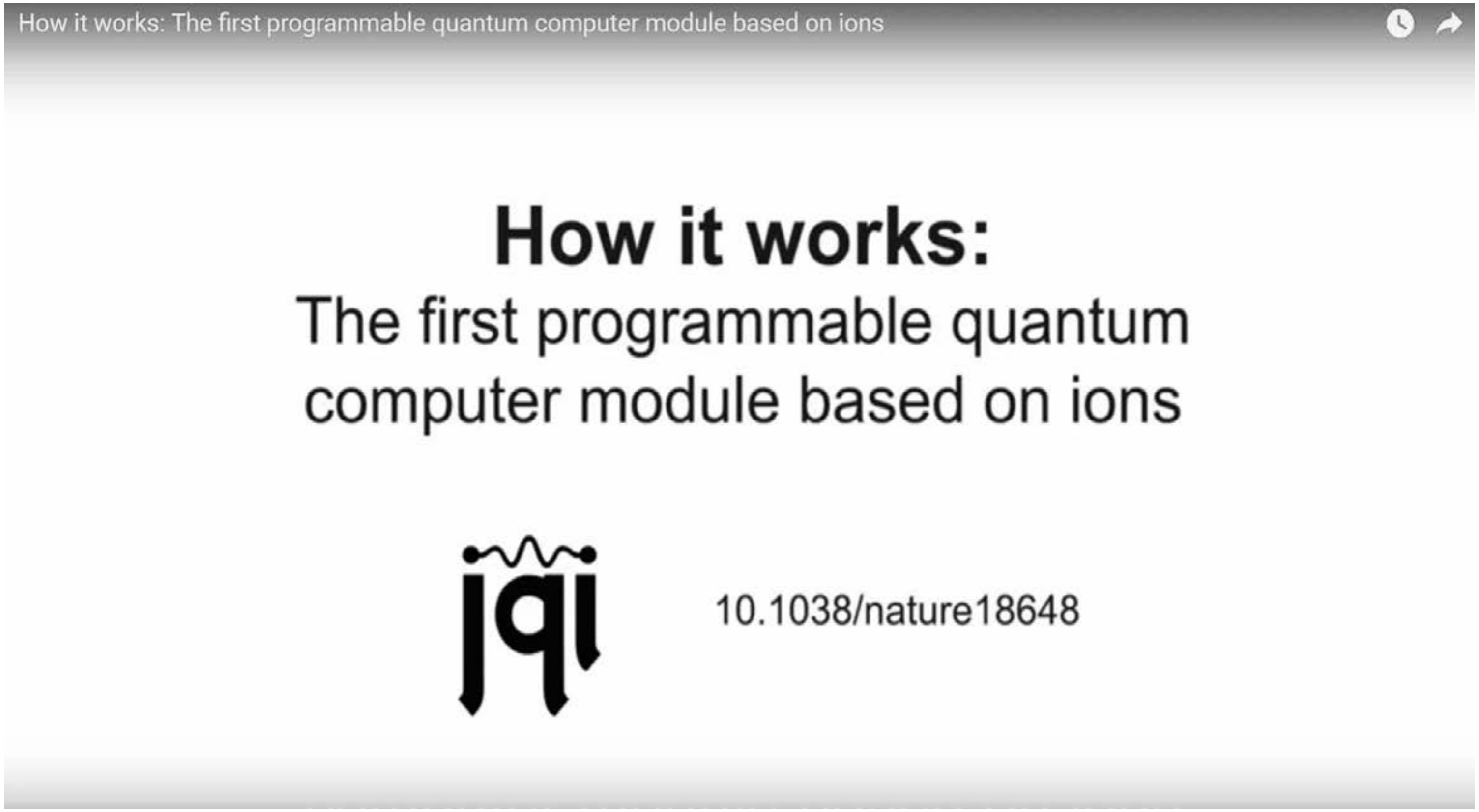
Ability to Extract a Final Measurement From The State of the Qubits at the Conclusion of the Quantum Computing Program

Measuring the Final State of the Two Qubit System for the $^{40}Ca^+$ Trapped Ion Quantum Computer

- Measurement is done using the 397 nm laser to detect whether or not there is fluorescence between the $P_{1/2} \rightarrow S_{1/2}$ transition
- If the ion is in the ground state (“0” state) then the ion will fluoresce and a 397 nm light signal will be observed
- If the ion is in the $D_{5/2}$ state (“1” state) there will be no fluoresce at 397 nm and no light signal will be observed



Ion Trap Quantum Computer Simulation*


How it works: The first programmable quantum computer module based on ions

How it works:

The first programmable quantum computer module based on ions

10.1038/nature18648

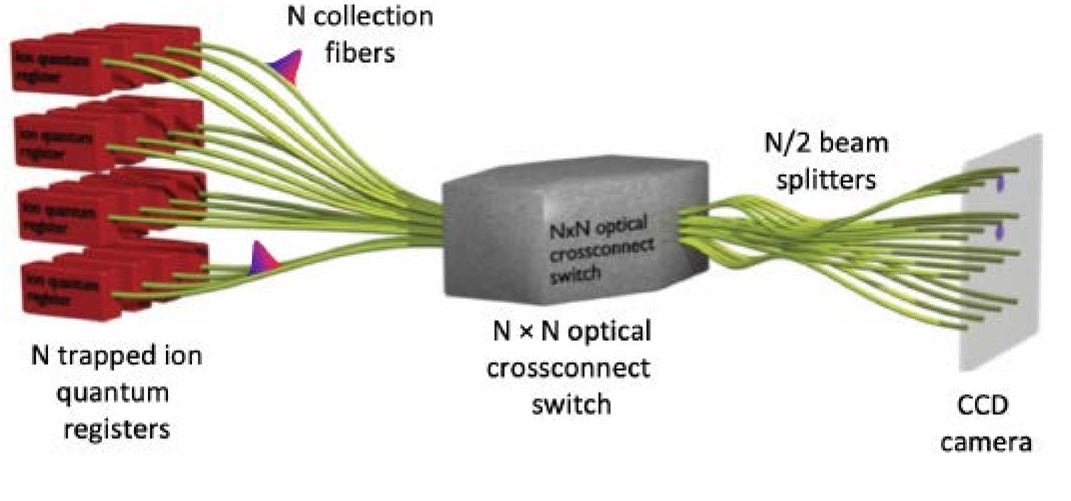
* https://www.youtube.com/watch?v=eK6g6ozLcVA

Step 4
System Must Be Scalable

Comments - Ongoing Research
Criteria 5 - System Must Be Scalable

- TIQC requires very pure state initialized which implies very low (milli-Kelvin) operating temperature for the apparatus
- The frequency of the data bus must be slower than the frequency of the center of mass phonon vibrational mode
- As the number of ions increases the difficulty of maintaining a coherent state wavefunction also increases (ex. stray external EM fields) – increasing likelihood of a destroying the coherence and leaving a collapsed wavefunction before the completion of the full set of gate operations
- Ongoing work to improve the performance and operation of TIQC devices

Scaled Design for a TIQC*



* Monroe, et.al "Large Scale Modular Quantum Computer Architecture with Atomic Memory and Photonic Interconnects", Phys. Rev. A 89, 022317 (2014)

Questions